# Is Geopolitical Risk a Genuine Factor of the Financial Market?

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## Abstract

We investigate empirically that if Geopolitical Risk can be counted as a true risk factor via a protocol for factor identification proposed by Pukthuanthong, Roll, and Subrahmanyam[2017]. Using the Geopolitical Risk indexes created by Caladara and Iacoviello[2018], we believe that none of the Geopolitical indexes(i.e., Geopolitical Action risk(GPA), Geopolitical Threat Risk(GPT) and the general Geopolitical Risk(GPR) significantly induce changes in the financial market. Furthermore, as we conduct a cross-sectional multiple regression analysis between geopolitical risk candidate factors and individual stock returns, we find that there is no evidence suggesting that GPR is priced into the market even though GPR's Sharpe ratio passes our predetermined threshold.

## 1. Introduction

In recent years, more and more investors worry about the economic impact of geopolitical events, such as wars, terrorist acts, or conflicts happening around the world and rank geopolitical risk as one of the primary concerns alongside both political and economic uncertainty. In general, such geopolitical events pose risks that fill the financial markets. However, despite rising awareness of geopolitical risk, our understanding of its effects on the economy and financial markets remains shallow. A potential reason is that there is no current research about whether geopolitical risk is a true factor to the market. Thus, the contribution of this paper can be summarized as follows: performing a protocol to conclude whether geopolitical risk is a genuine factor and comparing across the GPR, GPT, and GPA indices to examine which geopolitical risk factor induces changes in the market the most.

Kristof [1960] mentions that Geopolitics is a term that is notoriously difficult to define. Tuathail and Agnew [1992] points out that, in conventional academic understanding, geopolitics concerns the geography of international politics, particularly the relationship between the physical environment and the conduct of foreign policy. However, in recent decades, many other events, such as power struggles, have been classified as part of geopolitics. Thus, the word "geopolitics" covers a diverse set of events ranging from terrorist attacks to climate change and Brexit to the global Financial Crisis.

There have been several studies supporting the fact that geopolitical events, such as wars and terrorism, have significant effects on the economy and investments. Blomberg et al. [2004] suggest that the occurrences of terrorism harms the real economy badly and redirects economic activity away from investment. Schneider, Gerald, and Troeger [2006] also point out that political conflict has a negative impact on the core financial markets in the Western world.

Thus, Caldara and Iacoviello [2018], hereafter referred to as CI, have constructed a measure called the Geopolitical Risk (GPR) Index. It describes the risks associated with wars, terrorist acts, and tensions between nations that affect the normal course of international relations. Market participants, central bank officials, and many investors have gradually considered geopolitical risk (GPR) to be a key determinant of investment decisions and stock market dynamics. And GPR is believed to have an impact on business cycles and financial markets. In a 2017 Gallup survey, nearly 75% of respondents out of over 1,000 participants ranked geopolitical risk ahead of political and economic uncertainty, showing concerns over the economic impact of rising geopolitical risk. And CI finds that as geopolitical risk increases, stock returns decrease, and capital flows from emerging economies towards developed ones. To isolate the impacts of pure geopolitical risk, CI creates two variations: geopolitical threats index (GPT) and geopolitical acts index (GPA). The GPT is associated with information that directly mentions risks, and GPA searches for the words directly connected to adverse events.

In 2018, Aguilar and McCallen [2018], herein referred to as AM, investigates whether there is a relationship between GPR and U.S. equity returns. It turns out that GPA is comparatively related to returns and traditional equity risk factors. However, only the expected GPA seems to be priced, while others are not. Thus, we want to examine further if GPR is a risk factor in the financial market.

We draw on the methodology pioneered by Pukthuanthong, Roll, and Subrahmanyam [2017], herein referred to as PRS, and run the GPR through the proposed protocol, which states that the empirically measurable candidates must be related to the principal components of the covariance matrix to be considered a factor. Although these factors have received increasing attention as some of the most prevalent determinants in evaluating investment performance and

risk, there are no agreements to classify priced and non-priced factors. Thus, roll protocol can be viewed as a process used to identify factors that will be widely accepted academically.

There are two major stages for the roll protocol. The first stage of our paper assesses whether geopolitical risk induces change in the financial market. The first step is to compute the asymptotic principal components as suggested by Connor and Korajczk [1988]. The nonstationary is especially important for GPR since the GPR is event driven. To capture this characteristic, we compute the rolling ten-year window from 1989 to 2016 for the asymptotic principal components. Unlike Roll's approach of selecting the first 10 principal components, we set the number of principal components selected as dynamic and with a universal cutoff line at 90% explanatory power. We then proceed to take the canonical correlation between assets and factors. Due to high correlation among GPR, GPT, and GPA, we compute the Fama five factors and all three of the geopolitical indices separately instead of building a factor model, which avoids repetition when evaluating the canonical variates.

The first stage gives us important insights about whether certain factors induce changes in the market. Taking Roll's 2018 approach of deriving significance, we compute the weighted average principal components and run a regression using the weighted-average PC and all six factors as independent variables. Although GPR, GPT, and GPA indices do not pass the cut-off line, RMW and CMA match well with PRS's empirical analysis. The higher value of GPA relative to GPR and GPT also confirms AM's findings. For the first stage of the protocol, risk-free, SMB, HML, RMW, and CMA induce changes in the market, while geopolitical risk indices fail to pass the first step.

The second stage is to perform a cross-sectional panel regression to test whether the factor is priced in. Even though PRS suggests that only the factors that pass the first necessary condition

can proceed to the panel regression, we choose to examine the GPA index as well since the two stages are not interconnected. We compute Beta, also known as the OLS slope coefficient, based on the cross-sectional multiple regression and assign them to each candidate factor and the estimated risk premium for all factor candidates. The result suggests that risk-free, SMB, HML, and CMA pass the second stage. However, there seems to be a lack of correlation between GPA and individual equity, which eventually leads to insignificant risk premium.

The last step of the protocol will be to check whether the candidate factor has a reasonable range of return over volatility. To answer this question, we construct a hedge portfolio with a long position in the stocks and compute the Sharpe ratio. We then perform a t-test of the Sharpe ratio against the 0.6 threshold suggested by Mackinlay [1995].

Our paper is not the first academic literature examining the impact of geopolitical risk on financial markets. These works tend to be case studies that focus on specific events. To the best of our knowledge, we are the first to apply roll protocol to the geopolitical risk to determine if it is a factor, which is one step further than AM since they only did the second stage.

This paper presents several main findings. First, the significant levels of the canonical correlation prove that none of Geopolitical Action Risk (GPA), Geopolitical Threat Risk (GPT) and general Geopolitical Risk (GPR) significantly induces changes in the financial market. Second, the contribution of geopolitical risks to the explanatory power of the regression model is not significant. In other words, given the current Fama-French 5 factor model, there is no evidence showing that geopolitical risk is priced in to the market. Third, geopolitical risk do have a reasonable hedged portfolio return according to its Sharpe ratio. However, the protocol proposed by PRS requires a genuine factor to pass all three tests and geopolitical risk only satisfied one of them. We cannot classify geopolitical risk as a factor based on the protocol.

The rest of the paper proceeds as follows. Section 2 provides a literature review on those regarding geopolitical risk and roll protocol. Section 3 describes our data, which includes the CI GPR indexes and equity returns. Section 4 illustrates the detailed methodology we use for the paper. In Section 5, we provide all the empirical analyses and address questions, such as "Does geopolitical risk induce changes in the financial market?" The conclusion is found in Section 6.

## 2. Literature Review

Several studies show that geopolitical events, such as wars and terrorism, have significant effects on the economy and investments. Blomberg et al. [2014] find that the incidence of terrorism has an economically notable hit on the real economy and redirect the economy activity away from investment spending. Schneider, Gerald, and Troeger [2006] suggest that the conflicts affect the interaction at the core financial markets in the Western world negatively.

Caldara and Iacoviello [2018], herein referred to as CI, examines the influence of geopolitical risk on several global stock market indexes. According to CI, most of the respondents ranked geopolitical risk ahead of political and economic uncertainty and expressed concerns about the potential economic impact of rising geopolitical risk in a Gallup 2017 survey. Thus, CI constructs a monthly index of geopolitical risk (GPR) and examines its development and determinants since 1985 using newspaper records.

And it turns out that, for the U.S. stock market, industry sectors have various responses to the GPR index, with the defense sector showing positive excess returns while other sectors that are more connected to the more macroeconomy faced negative yield.

Currently, even though there are various indicators of political and geopolitical uncertainty developed by many companies, they are not a good fit for empirical analysis due to several limitations. First, the indexes from these companies do not clearly define geopolitical risk. Second, existing indexes are often not publicly available so that they are difficult to replicate. Third, many of these indexes are qualitative indicators with little variability.

On the other hand, CI believes that GPR is the first index to isolate pure movements at risk and does not require a strong identification assumption. CI defines geopolitical risk as to the risk associated with wars, terrorist acts, and tensions between states that affect the healthy and peaceful course of international relations. To further examine the impacts of pure geopolitical risk, CI hence creates two more variations, geopolitical threats index (GPT) and geopolitical acts index(GPA). While GPT is associated with the information that directly mentions risks, GPA searches for the words directly connect to adverse events. CI also suggests that GPR index gives more time variation, which allows relatively short samples to estimate the effects of geopolitical events on equity returns.

CI uses Bayesian techniques to estimate VAR models and a Cholesky decomposition of the covariance of the VAR reduced-form residuals to identify the structural shocks. And it turns out that a GPA shock will cause a small and temporary decline in economic activity. The stock market will quickly recover from it. On the other hand, a GPT shock will leave extensive and prolonged recessionary effects on the economy. And the results from CI indicate that exogenous changes in geopolitical risks slow down economic activity and upset the stock market due to threats of adverse geopolitical events rather than realizations.

In 2018, Aguilar, McCallen [2018], herein referred to as AM, tries to investigate if there is a relationship between GPR and U.S. equity returns by broadening the scope to a more significant cross-section, encompassing the entire CRSP universe.

To avoid idiosyncratic noise in individual returns, AM sorts all securities in the CRSP universe into five portfolios according to their sizes and five according to their Book-to-Market value. Thus, there are a total of 25 portfolios, which AM investigates with the traditional five sources of equity risk inspired by factor pricing models, the market risk premium (Mkt-Rf), market capitalization (SMB), book-to-market (HML), profitability (CMA), and investment (RMW), plus the GPR factor. It turns out that the GPR factor is less correlated with the five factors than they are with each other, which is not surprising since only GPR is not tradeable, based on the simple bivariate correlation test of the traditional risk factors with the GPR.

AM then, as suggested by CI, models the GPR as a simple AR(1) and takes the estimated value as the "expected" level of geopolitical risk, and the estimated residual as "shocks." Based on the result, AM points out that geopolitical risk seems more connected to the five factors than expected. GPR shocks seem to have a significant negative relationship with returns. Threats are unrelated to traditional equity risk factors. And the traditional risk factors can explain geopolitical actions. In the end, AM explores whether threats or actions can add explanatory power to the traditional five factors equity pricing model and finds that GPT is negatively related to returns across all portfolios examined while GPA is positively related.

Thus, AM concludes that GPA, different than GPT, is more closely related to returns and the traditional equity risk factors. While shocks and expected GPT are not priced, the expected GPA is priced with a similar magnitude as in the previous findings. It is worth to mention that since AM follows the two-pass methodology of Fama and Macbeth[1973], herein referred to as

FRM, an Errors-In-Variables problem occurs. The actual value tends to be overstated by large estimates of the price of risk and to be understated by small estimates.

In 2017, Pukthuanthong, Roll, and Subrahmanyam[2017], herein referred to as PRS, proposes a protocol for identifying genuine risk factors, assuming that a risk factor must be related to the covariance matrix of returns, priced in the cross-section of turns, and should yield a reward-to-risk that should be consistent with risk pricing. Factors have been considered one of the most prevalent determinants in analyzing investment performance and risk. PRS feels the necessity to create a process to identify factors that will be widely accepted academically since there has been no protocol suggested before to classify priced factors and non-priced factors separately.

Thus, PRS determines that the empirically measurable candidate must be related to the principal components of the covariance matrix in order to be considered as a factor.

However, factor extraction from the covariance matrix faces serious difficulties: firstly, it only provides estimates for the linear combinations of the underlying factors; secondly, since there is no reasoning for why the number of factors should be constant over time, we will suffer from non-stationarity. Third, factor extraction includes true risk drivers, pervasive non-diversifiable factors as well as some diversifiable factors that are not related to risk premiums.

Fortunately, PRS found cures for each of these conundrums even though they all have minor shortages. For (a), the linear combinations extracted by PCA could be related to other candidate factors through canonical correlation to have some reassurance. For (b), PCAs could be estimated for subperiods. For (c), a second method as in Fama and Macbeth(1973) could be employed to distinguish priced (presumably non-diversifiable) factors from others.

PRS further examines what counts as underlying factors and takes several approaches into account to identify and measure the factors. The first approach uses principal components or factor

analysis, (e.g., Roll and Ross, 1980; Connor and Korajczyk, 1988.) The second approach pre-specifies macro-economic variables that seem likely to be pervasive and then pre-whitens the official numbers of such low-frequency constructs(e.g., Chen, Roll and Ross, 1986.) And the last one relies on asset characteristics to develop proxies(e.g., Fama and French, 1993.)

Given the information above, PRS outlines the first stage of the roll protocol that identifies factors that move asset prices consistently. It is worth to notice that PRS does not differentiate factors with risk premium and factors that are not related to risk premiums until a later stage.

Firstly, PRS collects a set of equities from various industries and then extract principal components from the return series. As Connor and Korajczyk(1988), herein referred to as CK, suggest that for a large set of equities, analyzing the eigenvectors of $\Omega t$ is almost the same as factor analysis.

Then, PRS collects a set of factor candidates and uses the eigenvectors to calculate the covariance matrix. These factor candidates can be well-known characteristics based candidates, or others mentioned by PRS.

The last step for stage one is to compute the canonical correlations and transform these correlations into a variable. In this way, PRS can test whether the factor candidates are conditionally related to the covariance matrix of real returns as a group and identify factor candidates that seem to be more related to the real return covariances. PRS only looks for a strong canonical correlation between the linear combination of the factor candidates and the linear combination of eigenvectors. Any weak canonical correlation can be rejected.

In stage two, PRS examines if the risk factor is priced. PRS warns that all the procedures in this stage should be done with individual real asset returns rather than the portfolio returns. Since the regression betas are unknown quantities and must be estimated, it will cause a classic

errors-in-variables(EIV). The error variances for individual assets are almost certainly greater than they are in betas estimate for portfolios, which explains why Fama and French(1992) used the latter. Therefore, PRS adopts Fama and French's procedure in using portfolios to obtain beta estimates, assigning portfolio betas to the individual constituent stocks, and then checking to see if the factor is priced via FM regressions.

Besides, since Fama and French(1993) do not perform this second stage exercise on individual securities for their SMB and HML factors, PRS does a double sort and assigns the relevant portfolio betas on stocks.

As the final check following the FM regressions, PRS suggests that for a genuine risk factor, its reward-to-risk ratio must have reasonable limits and thus proposes a test to check if each factor passes the Mackinlay(1995) bound.

At the end, PRS test candidate factors, which include the five Fama-French(2015) market factors, factors proposed by the Carhart(1997), and Chen, Roll, and Ross(1986) factors. PRS then constructs the traded liquidity factor and Chen, Roll, and Ross(1986) 's factors using Cooper and Priestley(2011, henceforth CP) 's methodology, through the suggested protocol using simultaneous monthly return observations over a half-century. And it turns out that almost all characteristics are with significant premiums statistically, but only momentum and the book/market anomaly yield Sharpe ratios exceed the Mackinlay bound and are thus considered to abnormal profit opportunity. PRS also notes that there is no major difference between correcting for EIV and not doing so.

## 3. Data Description

### 3.1 GPR Data

Caldara and Iacoviello[2018], herein referred to as CI, constructs a monthly index of geopolitical risk since 1985. It is defined by CI as the risk associated with wars, terrorist acts, and tensions between states that affect the normal and peaceful course of international relations. The monthly index is calculated by dividing the number of articles regarding geopolitical risks by the number of total published articles each month via an automated text-searches of 11 newspapers electronic archives. These 11 newspapers are The Boston Globe, the Chicago Tribune, The Daily Telegraph, the Financial Times, The Globe and Mail, The Guardian, the Los Angeles Times, The New York Times, The Times, The Wall Street Journal, and The Washington Post. The index is normalized to average a value of 100 in the 2000-09 decade.

While CI identifies articles with references, they sort the keywords into six groups: Group 1 includes words associated with explicit mentions of geopolitical risk and military related tensions around the world. Group 2 includes words directly related to nuclear tensions. Groups 3 and 4 include mentions related to war threats and terrorist threats, respectively. Groups 5 and 6 include press coverage of actual adverse geopolitical events (as opposed to just risks).

Since risk often spikes with the realization of significant events, CI attempts to isolate the effects of pure geopolitical risk by constructing two indexes: the geopolitical threats index(GPT) and the geopolitical acts(GPA) index. The GPT index searches for groups directly mentioning risks while the GPA index searches the ones directly mentioning adverse events. GPT index includes words belonging to Search groups 1 to 4, and the GPA index includes words from groups 5 to 6.

Our study uses data from January 1986 through December 2016. The data is directly obtained from the CI's website. It is worth mentioning that since there has significantly more newspaper listing after 1985, we choose to begin our analysis on the period after 1985(i.e.,1986).

We also compute the correlation between GPR and the 48 Industry portfolios obtained from Ken French's data library for every ten years to examine if GPR has some connections with specific sectors. From 1985 to 1995, Soda, Comps, and LabEq were the top three sectors that were most correlated to GPR with 0.119, 0.0653, and 0.0551, respectively. And Meals, Cloths, and Textiles were the top three sectors that are least correlated to GPR with -0.441, -0.434, and -0.446 respectively. From 1995 to 2005, Soda, Guns, and Hlth were the top three sectors that were most correlated to GPR with 0.584, 0.558, and 0.503, respectively. And Autos, EleEq, and Boxes were the top three sectors that are least correlated to GPR with -0.434, -0.345, and -0.178 respectively. From 2005 to 2015, PerSv, Chips, and Banks were the top three sectors that were most correlated to GPR with 0.533, 0.512, and 0.496, respectively. And Gold, Beer, Coal were the top three sectors that were least correlated to GPR with -0.381, -0.179, and -0.102 respectively. From 2015 to 2019, Fun, BusSv, and Guns were the top three sectors that were most correlated to GPR with 0.605, 0.576, and 0.572, respectively. And Toys, Txtls, and Agric were the top three sectors that were least correlated to GPR with -0.445, -0.457, and -0.334 respectively. Overall, Fin, Guns, and Soda seems to be most positively correlated to GPR while Agric, Txtls, and Beer are least correlated to GPR.

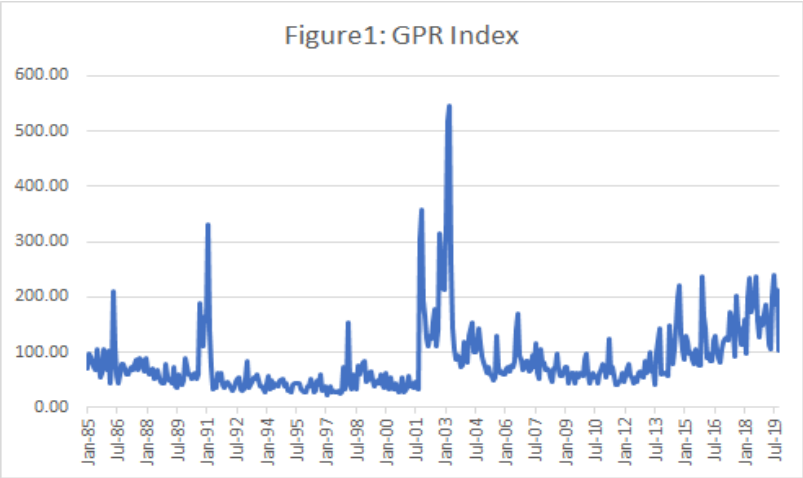
Similarly, we apply the same procedure to GPA and GPT. For GPT, from 1985 to 1995, Soda, Comps, and RlEst were the top three sectors that were most correlated to GPT with 0.0676, -0.0119, -0.0188 respectively. And Meals, Cloths, and LabEq were the top three sectors that are least correlated to GPT with -0.418, -0.409, and -0.397 respectively. From 1995 to 2005, Soda,

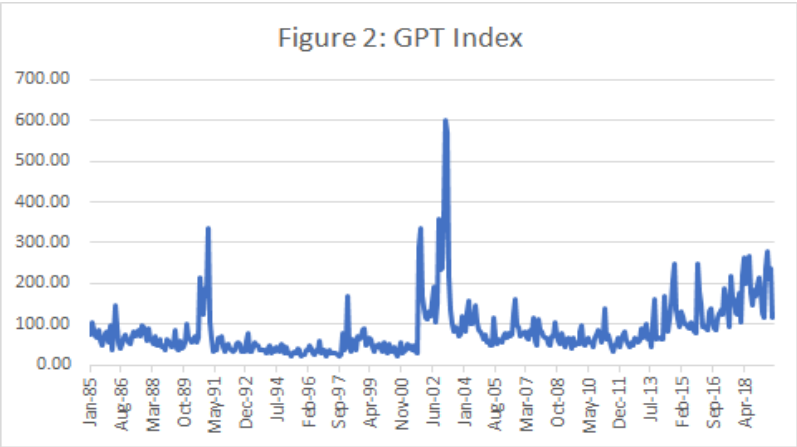
Guns, and Hlth were the top three sectors that were most correlated to GPT with 0.585, 0.565, and 0.485, respectively. And Autos, EleEq, and Boxes were the top three sectors that were least correlated to GPT with -0.410, -0.331, and -0.161 respectively. From 2005 to 2015, PerSv, Chips, and Guns were the top three sectors that were most correlated to GPT with 0.570, 0.560, and 0.526, respectively. And Gold, Beer, and Coal were the top three sectors that were least correlated to GPT with -0.408, -0.182, and -0.139, respectively. From 2015 to 2019, Fun, BusSv, and Guns were the top three sectors that were most correlated to GPT with 0.661, 0.633, and 0.630. And Txtls, Toys, and Agric were the top three sectors that were least correlated to GPT with -0.489, -0.479, and -0.389 respectively. Overall, Guns, Fin, and Soda are the sectors that are most correlated to GPT while Agric, Txtls, and Books are least correlated to GPT.

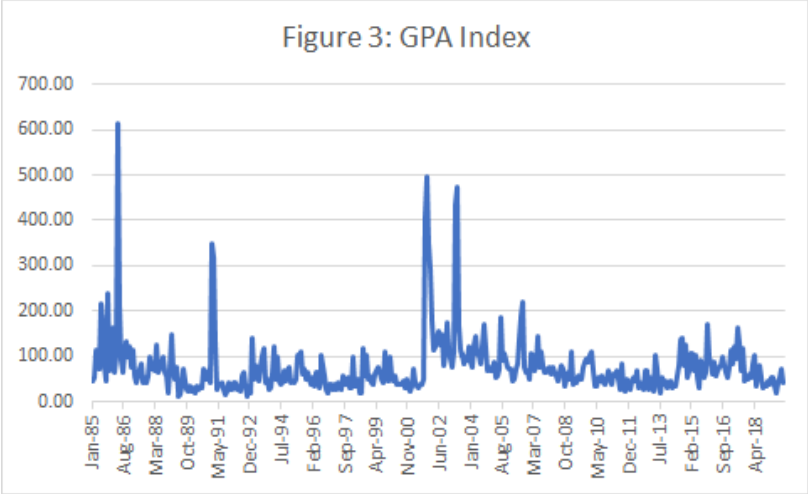
For GPA, from 1985 to 1995, Fun, BusSv, and Guns were the top three sectors that were most correlated to GPA with 0.661, 0.633, and 0.630, respectively. And Txtls, Toys, and Agric were the three sectors that were least correlated to GPA with -0.489, -0.479, and -0.389 respectively. From 1995 to 2005, Hlth, Soda, and Food were the top three sectors that were most correlated to GPA with 0.457, 0.399, and 0.398, respectively. And Autos, ElcEq, and Agric were the top three sectors that were least correlated to -0.427, -0.321, and -0.256 respectively. From 2005 to 2015, Books, Fun, and Ships were the top three sectors that were most correlated to GPA with 0.421, 0.336, and 0.320, respectively. And Comps, FabPr, and Agric were the top three sectors that were least correlated to GPA with -0.342, -0.274, and -0.220 respectively. From 2015 to 2019, Agric, Txtls, and Toys were the top three factors that were most correlated to GPA with 0.470, 0.419, and 0.247, respectively. And Clths, Hlth, and Rtail were the top three sectors that were least correlated to GPA with -0.520, -0.497, and -0.492 respectively. Overall, Fin, Ships, and Cnstr

seemed to be the most correlated sectors to GPA from 1985 to 2019 while Auto, Agric, and Rubbr were the least correlated sectors to GPA.

Last but not least, we calculate the correlation among GPR, GPT, and GPA for every ten years to investigate the relationship among these three indexes, as shown in the following graphs.






Note: Figure1, 2, and 3: depicts the time series of the GPR index in panel A, the GPT in panel B and GPA in panel C. Note the time series variability of each index and the apparent similarity across the indexes.

### 3.2 Equity Data

As the primary goal of our paper is to determine whether GPR displays a significant canonical correlation with its associated best linear combination of eigenvectors as viable factors, we need to decide the size of equities and periods, and how to address missing data.

PRS suggests that stage two should be done with individual real asset returns rather than portfolio returns due to several concerns. The first concern is that portfolios might hide features of individual assets, as Roll(1977) argues that the portfolio formation process can hardly reject the null hypothesis of no effects on security returns. Secondly, as Lewellen, Nagel, and Shanken(2010) caution, using portfolios decreases test power because there are fewer observations than individual assets. Thirdly, the analogous line fitted to portfolio returns and betas shows much smaller errors because once individual assets are grouped into portfolios and sorted by portfolio beta, the

individual errors are not related to beta anymore. Lastly, the choice of test portfolios significantly affects the statistical significance and economic magnitudes of risk premiums.

Hence, we begin our data creation by searching monthly individual stocks in the entire database from CRSP and include the holding period return without dividend as the query variable. We choose monthly updating to match with the GPR index. Note that we use data starting in 1986 due to concerns about the inconsistency of the GPR index for the first year. We delete observations that have missing values. To make sure the sample size is large enough to run the protocol, we have developed several designs to clean the data.

We split the overall sample into three ten-year-span subsamples first and do rolling subsamples as well to test the non-stationarity and to get rid of unwanted noises. We delete the stock data in 2018 and part of 2017 because there are too many missing values. The rolling subsamples include stock data from 1986-1996 to 2006-2016 as well as other samples that are less than ten years. Different than the five traditional risk factors, GPR is priced at some time in our ten years while sometimes not. We can detect when GPR is priced via using rolling data.

It is also worth mentioning that one drawback of using individual assets is the errors-in-variables(EIV) problem. The error variances for individual assets are almost certainly more significant than those in beta estimates for portfolios. Thus, we strictly follow PRS's approach to do a double sorting, assign the relevant betas on stocks, and check if the factor is priced via FM regressions. The beta estimates are obtained by using portfolios as suggested by Fama and French(1992).

## 4. Methodology

This section illustrates the methodology for our research project. It involves two stages: the first stage provides steps for testing necessary conditions for factor candidates to be valid, and the second stage whether factor candidates are diversifiable and priced.

### 4.1 Asymptotic Principal Component and Canonical Correlation

This stage identifies the factors that drive asset prices systematically. The following is the steps we apply for stage one.

First, we collect a set of N equities from the entire CRSP universe for the factor candidates to explain.

Second, we extract L principal components from the return series. The asymptotic approach of Connor and Korajczyk(CK)(1988) proposes TxT matrix $\Omega_t$=(1/N) RR', where R is the return vector. Based on two assumptions, the average squared beta converges (as n → infinite) to some value and the average $\varepsilon_i$ converges to $\sigma^2$, as n → ∞, $\Omega^n$ converges to:

$$\Omega = \bar{b}^2 \begin{bmatrix} (\gamma_1 + \tilde{f}_1)^2 & (\gamma_1 + \tilde{f}_1)(\gamma_2 + \tilde{f}_2) \\ (\gamma_1 + \tilde{f}_1)(\gamma_2 + \tilde{f}_2) & (\gamma_2 + \tilde{f}_2)^2 \end{bmatrix} + \sigma^2 I_2$$

Connor and Korajczyk(1986) suggests that as N increases, we will obtain consistent and asymptotically normal least squares estimates of the parameters.

However, PRS modifies the equation slightly by changing N to T. Thus, with T time-series units up to time t, the TxT matrix is now $\Omega_t$ =(1/T) RR'. PRS notes that since N is often greater than T, the characteristics of roll protocol allows us to use smaller-dimension TxT matrix $\Omega_t$, as opposed to the traditional NxN covariance matrix used for factor analysis.

Third, we collect a set of K factor candidates from Kenneth French's library and each of the geopolitical indices.

Fourth, we compute the canonical correlations between the factor candidates and the corresponding eigenvectors from the second step. The canonical correlation is the correlation between canonical variate pairs. And the canonical variates are the linear combinations of variables. Thus, we use the L eigenvectors and the K factor candidates to calculate the covariance matrix. The purpose of CCA is to find coefficient vectors $a_1 = (a_{11}, a_{21}, \dots, ap_1)^T$ and $b_1 = (b_{11}, b_{21}, \dots, bq_1)^T$ to maximize the correlation $\rho = \mathrm{corr}(Xa_1, Yb_1)$.

The two groups of multidimensional variables X and Y are equal to$x_i = \begin{bmatrix} x_{i1} \\ \cdots \\ x_{in} \end{bmatrix}$ and $y_i = \begin{bmatrix} y_{i1} \\ \cdots \\ y_{in} \end{bmatrix}$

Note that $U_1 = Xa_1$ and $V_1 = Yb_1$, i.e., linear combinations of X and Y respectively, are the first pair of canonical variates. Any factor candidate that does not show a significant canonical correlation can be rejected as a viable factor.

### 4.2 Panel Regression and Hedge Portfolio

The second stage of the protocol is to check if the candidate factor is priced in under FM regression. We adopt Fama and French (1992)'s procedure in using portfolios to obtain estimated betas and perform a multiple regression cross-sectionally to get the risk premium. Since PRS notes that there is no major difference between correcting for EIV and not doing so. We skip the double sorting and advance to the next step.

As a final check following the FM regression, we test if our risk factors exceed certain reasonable limits. Our bound is proposed by Mackinlay (1995). He argues that based on the historical mean excess and volatility of the CRSP value-weighted index, a reasonable annualized Sharpe ratio for a risk factor is 0.6 (corresponding for example, to an annualized excess return of

10% and a standard deviation of 16%). Thus, we test if our proposed factors have a Sharpe ratio is statistically higher than 0.6.

## 5. An Empirical Analysis

This section presents the methodology and results of using the protocol suggested by Roll (2018) to test whether the GPR index is a true factor. We used the sample assets from individual US equities listed on CRSP, 1986 - 2017 inclusive. Besides the GPR index, we also include five Fama-French (2015) factors as candidate factor: Market Risk Free, SMB, HML, RMW and CMA. We obtain these Fama-French factors from Ken French's Library. The addition of these factors helps the model achieve a better explanatory power. All returns and factors are monthly, same as GPR index.

In summary, the Roll protocol suggests three steps to test whether a candidate factor is a true factor: whether the candidate factors reduces changes in asset prices, whether the candidate factor is priced into the market, and whether the candidate factor has a reasonable range of Sharpe Ratio.

### 5.1 Does geopolitical risk induce changes in the financial market?

The first step of the protocol is to compute the asymptotic principal components as suggested by Connor and Korajczk (1988) to capture the cross-sectional characters of CRSP individual stock return. In Roll (2018)'s research, it splits the overall sample into five subsamples with ten year each to mitigate the possible non-stationarity of the factors. The non-stationarity is especially important for GPR index since the geopolitical risk is event-driven, non-stationary is very significant during a certain period of time. To capture this characteristic of GPR index, instead

of the ten-year bucket method suggested by Roll (2018), we also compute the rolling ten-year window from 1989 to 2016 for the asymptotic principal components. For each 10-year window, we extract the principal components using Connor and Korajczk (1988) method.

**Table 1: a summary of the principal components for ten-year buckets.**

| | 1988-1997 | | | | 1998-2007 | | | | 2008-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Skewness | Max | Min | Mean | Skewness | Max | Min | Mean | Skewness | Max | Min |
| **PC1** | 0.02 | 0.05 | 0.32 | -0.20 | -0.01 | 0.23 | 0.23 | -0.17 | -0.01 | 0.51 | 0.33 | -0.21 |
| **PC2** | 0.01 | 2.74 | 0.66 | -0.26 | -0.01 | -0.01 | 0.25 | -0.23 | -0.01 | 0.46 | 0.36 | -0.29 |
| **PC3** | -0.01 | 0.24 | 0.26 | -0.19 | 0.01 | 0.48 | 0.32 | -0.23 | 0.01 | 1.60 | 0.52 | -0.23 |
| **PC4** | 0.01 | 0.44 | 0.28 | -0.19 | -0.02 | -0.12 | 0.29 | -0.27 | 0.01 | 0.47 | 0.31 | -0.20 |
| **PC5** | 0.00 | 0.32 | 0.28 | -0.18 | 0.01 | -0.08 | 0.21 | -0.21 | 0.01 | 0.12 | 0.29 | -0.22 |
| **PC6** | 0.00 | -0.10 | 0.20 | -0.23 | -0.01 | 0.18 | 0.20 | -0.20 | 0.00 | 0.49 | 0.29 | -0.20 |
| **PC7** | 0.01 | 1.07 | 0.43 | -0.24 | -0.01 | -0.28 | 0.24 | -0.29 | 0.01 | 0.12 | 0.32 | -0.28 |
| **PC8** | 0.01 | 1.10 | 0.43 | -0.24 | -0.01 | -0.20 | 0.24 | -0.33 | 0.00 | 0.07 | 0.21 | -0.24 |
| **PC9** | 0.00 | 0.42 | 0.34 | -0.22 | 0.00 | 0.11 | 0.30 | -0.30 | -0.01 | 0.09 | 0.35 | -0.25 |
| **PC10** | 0.00 | 0.54 | 0.35 | -0.27 | 0.00 | 0.44 | 0.41 | -0.26 | 0.01 | 0.82 | 0.39 | -0.31 |

The skewness for 1988-1997 and 2008-2017 tends to be positive skewed, while during 1998 to 2007, most of the principal components are negatively skewed. Though it might lead to the inaccuracy of the dataset, the absolute value of skewness is on average below 0.5. Unlike Roll's approach of selecting the first 10 principal components, we set the number of principal components selected as dynamic and varies for each year with a universal cut off line 90% explanatory power.

The following table shows the number of principal components required to achieve the 90% explanatory power for the whole asset sample. This table shows that the dispersion of the dataset varies across years, and to keep the same level of explanatory power, a rolling window with dynamic principal components can improve the accuracy.

**Table 2: Number of PCs required to achieve the 90% explanatory power over year**

| Number of PC required to achieve the 90% explanatory power over year | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
| 13 | 19 | 24 | 56 | 51 | 34 | 35 | 33 | 31 | 31 |
| | | | | | | | | | |
| 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| 28 | 25 | 26 | 25 | 26 | 30 | 28 | 28 | 26 | 24 |
| | | | | | | | | | |
| 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| 23 | 23 | 22 | 21 | 29 | 24 | 24 | 23 | 20 | 18 |
| | | | | | | | | | |
| 2016 | | | | | | | | | |
| 13 | | | | | | | | | |

The protocol then proceeds to the canonical correlation between assets and factors. Since we are trying to find the correlation both cross-sectionally and considering multiple factors. The canonical correlation first computes several pairs canonical variates that are combinations of weighted dependent variables (asset returns) and combinations of weighted factors as independent variables. For each pair, the canonical variates are orthogonal with a particular intercorrelation. We then sort these pairs from largest to smallest squared correlation since the correlation can be either negative or positive. The canonical pairs has min(L,K) possible pairs where L is the number of factors and K is the number of principal components we extracted. Here we have L = 6 and K = 10. Due to the high correlation among GPR, GPT, and GPA index, instead of building a factor model with Fama five factors and all three of the geopolitical indices, we compute them separately. We first group the Fama five factors with GPR and run through the canonical correlation, then Fama five factors with GPT, then GPA. This helps avoid repetition when evaluating the canonical variates.

**Table 3: Canonical correlation of Fama 5 Factors, GPR, GPT and GPA**

| Fama 5 Factor & GPR | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1988-1997 | | | 1998-2007 | | | 2008-2017 | | |
| | Correlation | tStat | pValue | Correlation | tStat | pValue | Correlation | tStat | pValue |
| Pair 1 | **0.95** | **7.28** | **0.00** | **0.97** | **9.25** | **0.00** | **0.96** | **6.38** | **0.00** |
| Pair 2 | **0.94** | **4.98** | **0.00** | **0.95** | **6.06** | **0.00** | **0.87** | **3.63** | **0.00** |
| Pair 3 | **0.77** | **2.62** | **0.00** | **0.83** | **3.39** | **0.00** | **0.75** | **2.33** | **0.00** |
| Pair 4 | **0.61** | **1.77** | **0.00** | **0.78** | **2.36** | **0.00** | **0.59** | **1.52** | **0.01** |
| Pair 5 | **0.51** | **1.46** | **0.04** | 0.56 | 1.08 | 0.32 | 0.51 | 1.14 | 0.23 |
| Pair 6 | 0.46 | 1.35 | 0.13 | 0.27 | 0.34 | 1.00 | 0.33 | 0.64 | 0.81 |
| Fama 5 Factor & Threat | | | | | | | | | |
| | 1988-1997 | | | 1998-2007 | | | 2008-2017 | | |
| | Correlation | tStat | pValue | Correlation | tStat | pValue | Correlation | tStat | pValue |
| Pair 1 | **0.95** | **7.15** | **0.00** | **0.97** | **9.23** | **0.00** | **0.96** | **6.38** | **0.00** |
| Pair 2 | **0.93** | **4.87** | **0.00** | **0.95** | **6.06** | **0.00** | **0.87** | **3.63** | **0.00** |
| Pair 3 | **0.77** | **2.55** | **0.00** | **0.83** | **3.39** | **0.00** | **0.75** | **2.33** | **0.00** |
| Pair 4 | **0.61** | **1.69** | **0.00** | **0.78** | **2.37** | **0.00** | **0.59** | **1.51** | **0.01** |
| Pair 5 | **0.51** | **1.34** | **0.08** | 0.56 | 1.08 | 0.32 | 0.51 | 1.13 | 0.24 |
| Pair 6 | 0.42 | 1.11 | 0.30 | 0.27 | 0.35 | 1.00 | 0.32 | 0.62 | 0.83 |
| Fama 5 Factor & Action | | | | | | | | | |
| | 1988-1997 | | | 1998-2007 | | | 2008-2017 | | |
| | Correlation | tStat | pValue | Correlation | tStat | pValue | Correlation | tStat | pValue |
| Pair 1 | **0.95** | **8.06** | **0.00** | **0.97** | **9.40** | **0.00** | **0.96** | **6.36** | **0.00** |
| Pair 2 | **0.93** | **5.69** | **0.00** | **0.95** | **6.16** | **0.00** | **0.87** | **3.63** | **0.00** |
| Pair 3 | **0.78** | **3.34** | **0.00** | **0.83** | **3.51** | **0.00** | **0.75** | **2.34** | **0.00** |
| Pair 4 | **0.70** | **2.64** | **0.00** | **0.78** | **2.47** | **0.00** | **0.59** | **1.53** | **0.01** |
| Pair 5 | **0.59** | **2.05** | **0.00** | 0.56 | 1.24 | 0.14 | 0.51 | 1.15 | 0.22 |
| Pair 6 | **0.51** | **1.76** | **0.03** | 0.35 | 0.64 | 0.85 | 0.33 | 0.65 | 0.80 |

Table x represents the results of canonical correlations between Fama five factors and GPR, GPT, and GPA, respectively. The canonical variates are sorted in descending order by their estimated squared T-Stats. This canonical correlation reveal that these six factors induce a higher degree of changes in 1988-1997 than the last decade. Further comparison between GPR, GPT, and GPA shows that the significant level of GPA is the highest across all years. This substantiates previous findings in Aguilar and McCallen (2018) that "Geopolitical actions...appear to be more closely related to returns and the traditional equity risk factors than are threats."

The results from canonical correlation only give us the significant level of a certain combination of factors. To assess the significance of certain factors and determine whether they induce changes in the market, Roll (2018)'s approach of deriving significance is used. First, we computed the weighted average principal components using the optimal weights indicated by the

canonical correlation, i.e. the U vector. Second, for each year, we run a regression using the weighted-average PC as the dependent variable and all six factors as independent variables. We repeat this process for all GPR, GPA and GPT. We then extract the t-stat and p-value for all years and compute the averaged T-Statistic for each factor. Roll (2018) suggests a critical rejection levels for T-Statistic at 1.96 (5%), and for the factors exceeding 1.96 T-Statistics are bold. The table below shows the mean T-Statistics from 1986 to 2017. Although all three of GPR, GPT and GPA indices don't pass the cut off line, the low T-Statistic for two commonly-known factors, RMW and CMA actually matches well with Roll [2018]'s empirical analysis and the higher value of GPA relative to GPR and GPT also confirms Aguilar and McCallen (2018)'s finding.

**Table 4: t-stat for Factor candidates**

| | Factor Candidates | | | | | |
|---|---|---|---|---|---|---|
| | RF | SMB | HML | RMW | CMA | GPR |
| Mean t-stat | **4.590** | **1.951** | **3.227** | 1.522 | 1.297 | 0.679 |
| | | | | | | |
| | RF | SMB | HML | RMW | CMA | GPT |
| Mean t-stat | **4.727** | **1.917** | **3.209** | 1.513 | 1.320 | 0.672 |
| | | | | | | |
| | RF | SMB | HML | RMW | CMA | GPA |
| Mean t-stat | **4.726** | **2.324** | **3.175** | 1.455 | 1.307 | 0.817 |

An alternative way to check whether the factor has significant canonical correlation is to count the number of times p value is smaller than 0.05 out of 10 year for each rolling decades. This approach is similar to Roll (2018)'s, where they count the number of T-Statistics larger than 1.96 for each decade in the sample period. One possible drawback with this method is that the degree of freedom of the sample is dynamic and cannot be captured by a universal T-Statistics cutoff line. Instead, a rolling window using p-Value can better capture the diversity over time and in terms of the average counts, the two methods should produce similar results.

**Table 5: Number of p-Value <0.05 out of 10 for each decade**

| Number of p-Value < 0.05 out of 10 for each decade | | | | | | |
|---|---|---|---|---|---|---|
| | RF | SMB | HML | RMW | CMA | GPR |
| Mean | **4.452** | **4.000** | **4.194** | **4.032** | **3.677** | 1.548 |
| | | | | | | |
| | RF | SMB | HML | RMW | CMA | GPT |
| Mean | **4.387** | **4.000** | **4.226** | **3.968** | **3.710** | 1.419 |
| | | | | | | |
| | RF | SMB | HML | RMW | CMA | GPA |
| Mean | **4.645** | **4.065** | **4.452** | **4.161** | **3.839** | 2.258 |

Roll (2018) suggests a threshold of 2.5 to determine whether the factor passes the first step of this protocol. We are aware that this 2.5 threshold is arbitrary and its purpose is still unclear. It is plausible that they choose a midpoint between the minimum 0 and maximum 5 as the threshold. We decided to apply the same threshold to our model. The results show that risk-free, SMB, HML, RMW, and CMA induce changes in the market, i.e. passing the first step of the protocol, while none of the geopolitical risk indices is significant enough to pass the first step.

### 5.2 Does the factor priced in?

The second step of the protocol is to perform a cross-sectional panel regression to test whether the factor is priced in. According to the protocol suggested by Roll (2018), only the factor that pass the first necessary condition can be proceeded to the panel regression. However, since the two steps are not interconnected, we choose the geopolitical risk action index, the one that is most likely to pass among all geopolitical risk indices, as an additional factor in the second step. Beta, also known as the OLS slope coefficient, is computed based on the cross-sectional multiple regression among all individual stocks. The table below is the cross-section distribution of each Betas from 1996 to 2017 for all CRSP universe.

**Table 6: Summary Statistics for the Candidate Factor Betas and Characteristics**

| Summary Statistics for the Candidate Factor Betas and Characteristics | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | Median | Std | Skewness | Kurtosis | Maximum | Minimum |
| RF | -0.01 | -0.01 | 0.37 | 0.25 | 3.25 | 1.49 | -0.86 |
| SMB | 0.44 | 0.40 | 0.50 | 0.44 | 3.09 | 2.39 | -0.78 |
| HML | 0.28 | 0.27 | 0.52 | -0.02 | 4.16 | 2.30 | -1.78 |
| RMW | 0.28 | 0.33 | 0.49 | -1.69 | 10.24 | 1.67 | -3.31 |
| CMA | 0.14 | 0.17 | 0.46 | 0.21 | 6.88 | 3.18 | -1.81 |
| GPA | 0.00 | 0.00 | 0.01 | 0.29 | 6.91 | 0.04 | -0.04 |

The evidence from this table implies that GPA has a very small coefficient for the linear factor model. Besides, given its sample size, the kurtosis for GPA is very large, which implies a leptokurtic distribution. We then compute the estimated risk premium for all factor candidates. The risk premium is estimated from cross-sectional regression using individual stock returns from 1986 to 2017 as dependent variable and betas of six candidate factors as independent variables. We average over all month cross-sectional coefficient to construct risk premium.

**Table 7: Estimated Risk Premiums for Factor Candidates**

| Estimated Risk Premiums for Factor Candidates | | | |
|---|---|---|---|
| | Estimate | tStat | pValue |
| Constant | 0.43 | 10.74 | 0.00 |
| RF | 0.68 | 10.56 | 0.00 |
| SMB | 0.30 | 6.36 | 0.00 |
| HML | -0.39 | -7.38 | 0.00 |
| RMW | -0.02 | -0.37 | 0.71 |
| CMA | -0.12 | -2.06 | 0.04 |
| GPA | -1.19 | -0.39 | 0.69 |

Based on the results of the panel regressions, while the constant has the highest T-Stat, risk-free, SMB, HML, and CMA pass the second step of the protocol. The apparent lack of correlation between GPA and individual stock data can be attributed to the high kurtosis and low beta, which eventually lead to an insignificant risk premium.

### 5.3 Return of Hedge Portfolios

The last step of this protocol is to check whether the candidate factor has a reasonable range of return over volatility. For each candidate factor, we construct a hedge portfolio with a long position in the stocks with the top 10% of the beta and a short position in the stock with bottom 10% of the beta. We then compute the Sharpe Ratio and perform t-test of the Sharpe Ratio against the 0.6 threshold suggested by MacKinlay (1995). As expected, the t-test demonstrate that all candidate factors pass the third step and was within a reasonable range of Sharpe Ratio.

Table: Hedge portfolio from 10% top and bottom

**Table 8: Statistics for Return on Hedged Portfolios**

| | MRK-RF | SMB | HML | RMW | CMA | Action |
|---|---|---|---|---|---|---|
| **Mean** | 0.330 | 0.060 | -0.004 | 0.272 | 0.113 | 0.186 |
| **Std. Dev** | 4.050 | 3.695 | 3.810 | 2.611 | 2.245 | 2.745 |
| **Sharpe** | 0.081 | 0.016 | -0.001 | 0.104 | 0.050 | 0.068 |
| **Sharpe (t-stat)** | -10.083 | -11.351 | -11.685 | -9.641 | -10.684 | -10.351 |

## 6. Conclusion and Possible Extension

We have concluded an empirical study on whether geopolitical risk is a genuine factor to the financial market by applying the protocol proposed by Roll (2018). Geopolitical risk is an event-driven factor that present paltry market intrinsic value and thus it is necessary to incorporate a rolling window for the cross-sectional asymptotic principal components. The significant levels of the canonical correlation prove that none of Geopolitical Action Risk (GPA), Geopolitical Threat Risk (GPT) and general Geopolitical Risk (GPR) significantly induces changes in the financial market.

Focusing on whether geopolitical risk is priced in to the market, we conducted a cross-sectional multiple regression analysis between geopolitical risk candidate factors and individual stock return. The result shows that the contribution of geopolitical risks to the explanatory power

of the regression model is not significant. In other words, given the current Fama French 5 factor model, there is no evidence showing that geopolitical risk is priced in to the market.

We find that geopolitical risk does have a reasonable hedged portfolio return according to its Sharpe Ratio. However, the protocol proposed by Roll (2018) requires a genuine factor to pass all three tests and geopolitical risk only satisfied one of them. We cannot classify geopolitical risk as a factor based on the protocol. It is still unclear whether some inherent characteristics of geopolitical risk, such as threshold effect, event-driven, etc. might affect our results. To further our research, we intend to check if the geopolitical risk only got priced in within a certain range to discover the relationship between geopolitical risk and the financial market.

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# 8. Appendix:

**Table 9-11: Number of p-Value <0.05 for each rolling decade**

| Number of p-Value < 0.05 out of 10 for each rolling decade | | | | | | |
|---|---|---|---|---|---|---|
| Year/Factor | RF | SMB | HML | RMW | CMA | GPR |
| 1986 | 6 | 3 | 5 | 3 | 3 | 0 |
| 1987 | 6 | 4 | 4 | 3 | 1 | 3 |
| 1988 | 3 | 6 | 5 | 4 | 4 | 1 |
| 1989 | 6 | 5 | 4 | 5 | 3 | 3 |
| 1990 | 5 | 5 | 6 | 5 | 6 | 2 |
| 1991 | 5 | 4 | 6 | 5 | 4 | 5 |
| 1992 | 5 | 4 | 6 | 4 | 5 | 5 |
| 1993 | 5 | 3 | 5 | 3 | 5 | 4 |
| 1994 | 5 | 3 | 5 | 4 | 4 | 1 |
| 1995 | 5 | 4 | 5 | 5 | 4 | 1 |
| 1996 | 4 | 5 | 3 | 5 | 4 | 1 |
| 1997 | 4 | 6 | 4 | 5 | 4 | 1 |
| 1998 | 5 | 5 | 5 | 4 | 4 | 1 |
| 1999 | 5 | 3 | 5 | 5 | 4 | 1 |
| 2000 | 5 | 3 | 5 | 5 | 4 | 1 |
| 2001 | 4 | 4 | 4 | 5 | 4 | 1 |
| 2002 | 4 | 4 | 5 | 4 | 4 | 1 |
| 2003 | 5 | 3 | 2 | 5 | 4 | 1 |
| 2004 | 5 | 4 | 4 | 4 | 4 | 1 |
| 2005 | 5 | 4 | 3 | 3 | 2 | 1 |
| 2006 | 4 | 3 | 3 | 4 | 3 | 1 |
| 2007 | 4 | 3 | 3 | 3 | 2 | 1 |
| 2008 | 4 | 4 | 3 | 3 | 3 | 1 |
| 2009 | 4 | 5 | 5 | 2 | 2 | 1 |
| 2010 | 3 | 3 | 2 | 3 | 4 | 2 |
| 2011 | 3 | 3 | 4 | 2 | 3 | 1 |
| 2012 | 3 | 3 | 5 | 3 | 3 | 1 |
| 2013 | 3 | 4 | 4 | 4 | 4 | 1 |
| 2014 | 4 | 4 | 3 | 5 | 5 | 2 |
| 2015 | 3 | 4 | 3 | 5 | 4 | 2 |
| 2016 | 6 | 6 | 4 | 5 | 4 | 0 |
| Mean | 4.5 | 4.0 | 4.2 | 4.0 | 3.7 | 1.5 |

| Number of p-Value < 0.05 out of 10 for each rolling decade | | | | | | |
|---|---|---|---|---|---|---|
| Year/Factor | RF | SMB | HML | RMW | CMA | Threat |
| 1986 | 6 | 3 | 6 | 3 | 2 | 0 |
| 1987 | 6 | 4 | 4 | 2 | 3 | 1 |
| 1988 | 2 | 6 | 5 | 4 | 4 | 1 |
| 1989 | 6 | 5 | 4 | 5 | 3 | 2 |
| 1990 | 5 | 5 | 6 | 6 | 6 | 2 |
| 1991 | 5 | 4 | 6 | 5 | 4 | 5 |
| 1992 | 5 | 4 | 6 | 4 | 5 | 5 |
| 1993 | 5 | 3 | 5 | 3 | 5 | 4 |
| 1994 | 5 | 3 | 5 | 4 | 4 | 1 |
| 1995 | 5 | 4 | 5 | 5 | 4 | 1 |
| 1996 | 4 | 5 | 3 | 5 | 4 | 1 |
| 1997 | 4 | 6 | 3 | 5 | 4 | 1 |
| 1998 | 5 | 5 | 5 | 4 | 4 | 1 |
| 1999 | 5 | 3 | 5 | 5 | 4 | 1 |
| 2000 | 4 | 3 | 5 | 5 | 4 | 1 |
| 2001 | 4 | 4 | 4 | 5 | 4 | 1 |
| 2002 | 4 | 4 | 5 | 4 | 4 | 1 |
| 2003 | 5 | 3 | 2 | 5 | 4 | 1 |
| 2004 | 5 | 4 | 4 | 4 | 4 | 1 |
| 2005 | 5 | 4 | 3 | 3 | 2 | 1 |
| 2006 | 4 | 3 | 3 | 4 | 3 | 1 |
| 2007 | 4 | 3 | 3 | 3 | 2 | 1 |
| 2008 | 4 | 4 | 3 | 3 | 3 | 1 |
| 2009 | 4 | 5 | 5 | 2 | 2 | 1 |
| 2010 | 3 | 3 | 2 | 3 | 4 | 2 |
| 2011 | 3 | 3 | 4 | 2 | 3 | 1 |
| 2012 | 3 | 3 | 5 | 3 | 3 | 1 |
| 2013 | 3 | 4 | 5 | 4 | 4 | 1 |
| 2014 | 4 | 4 | 3 | 5 | 5 | 1 |
| 2015 | 3 | 4 | 3 | 4 | 3 | 2 |
| 2016 | 6 | 6 | 4 | 4 | 5 | 0 |
| Mean | 4.4 | 4.0 | 4.2 | 4.0 | 3.7 | 1.4 |

| Number of p-Value < 0.05 out of 10 for each rolling decade | | | | | | |
|---|---|---|---|---|---|---|
| Year/Factor | RF | SMB | HML | RMW | CMA | Action |
| 1986 | 6 | 4 | 4 | 3 | 3 | 1 |
| 1987 | 6 | 4 | 6 | 5 | 3 | 3 |
| 1988 | 3 | 6 | 4 | 5 | 3 | 4 |
| 1989 | 6 | 6 | 5 | 5 | 4 | 6 |
| 1990 | 6 | 5 | 6 | 6 | 5 | 6 |
| 1991 | 5 | 4 | 6 | 5 | 5 | 5 |
| 1992 | 5 | 4 | 6 | 4 | 5 | 5 |
| 1993 | 5 | 3 | 6 | 3 | 5 | 4 |
| 1994 | 6 | 4 | 6 | 5 | 5 | 5 |
| 1995 | 5 | 5 | 6 | 5 | 5 | 4 |
| 1996 | 5 | 3 | 3 | 4 | 4 | 1 |
| 1997 | 5 | 6 | 4 | 5 | 4 | 1 |
| 1998 | 6 | 4 | 5 | 4 | 4 | 1 |
| 1999 | 5 | 3 | 5 | 5 | 4 | 1 |
| 2000 | 4 | 3 | 5 | 5 | 4 | 1 |
| 2001 | 4 | 5 | 5 | 5 | 4 | 1 |
| 2002 | 4 | 4 | 6 | 5 | 5 | 3 |
| 2003 | 5 | 3 | 3 | 4 | 5 | 3 |
| 2004 | 5 | 4 | 5 | 4 | 4 | 1 |
| 2005 | 5 | 3 | 3 | 3 | 2 | 1 |
| 2006 | 4 | 3 | 3 | 4 | 3 | 1 |
| 2007 | 4 | 3 | 3 | 3 | 2 | 1 |
| 2008 | 4 | 3 | 3 | 3 | 3 | 1 |
| 2009 | 4 | 5 | 5 | 2 | 2 | 1 |
| 2010 | 3 | 3 | 2 | 3 | 4 | 2 |
| 2011 | 3 | 3 | 4 | 3 | 3 | 1 |
| 2012 | 4 | 3 | 5 | 4 | 4 | 1 |
| 2013 | 4 | 5 | 4 | 4 | 3 | 1 |
| 2014 | 4 | 5 | 3 | 4 | 5 | 2 |
| 2015 | 3 | 4 | 4 | 4 | 3 | 2 |
| 2016 | 6 | 6 | 3 | 5 | 4 | 0 |
| Mean | 4.6 | 4.1 | 4.5 | 4.2 | 3.8 | 2.3 |

**Figure 4: Return on Hedged Portfolio**

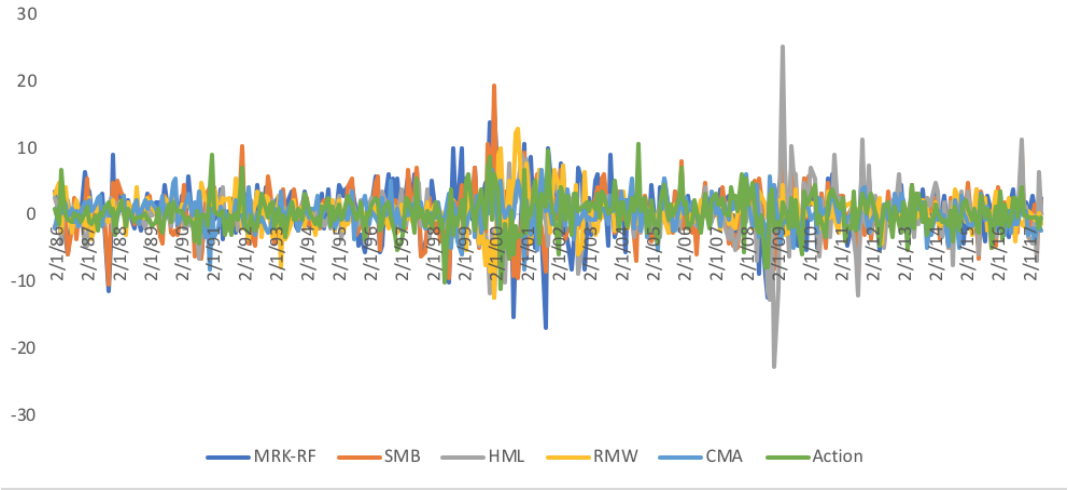


**Figure 5: T-Stat for all candidate factors for canonical correlation**