Multinational Expansion in Time and Space*

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October 2, 2019

Abstract

This paper studies the expansion patterns of the multinational enterprise (MNE) in time and space. Using a long panel of US MNEs, we document that MNE affiliates usually start with sales exclusively to the host market and eventually enter export markets, and that this extensive margin of expansion accounts for most of their sales growth. Informed by these facts, we develop a multi-country quantitative dynamic model of the MNE that features heterogeneity in firm-level productivity, persistent aggregate shocks, and a rich structure of costs that affect MNE expansion. Importantly, MNE affiliates can decouple their locations of production and sales, and endogenously choose to enter or exit the host and export markets. We introduce a novel compound option formulation that allows us to capture in a tractable way the rich heterogeneity observed in the data, which is necessary for quantitative analysis. Using the calibrated model, our quantitative application to Brexit reveals that the nature of the frictions to MNE activities matters for understanding the reallocation of MNE activity in time and space and for predicting the effects of globalization shocks.

JEL Codes: F1.

Key Words: Multinational firms, foreign direct investment, firm dynamics, sunk costs.

*We thank our discussants Javier Cravino, Ana Maria Santacreu, Michael Sposi, and Stephen Yeaple for very helpful comments in more than one occasion. We have also benefited from comments from George Alessandria, Costas Arkolakis, Ariel Burstein, Jonathan Eaton, Oleg Itskhoki, Sam Kortum, Andrei Levchenko, Eduardo Morales, Ezra Oberfield, and Andrès Rodríguez-Clare, as well as seminar participants at various conferences and institutions. Xiao Ma provided outstanding research assistance. The statistical analysis of firm-level data on US multinational companies was conducted at the Bureau of Economic Analysis, US Department of Commerce, under arrangements that maintain legal confidentiality requirements. The views expressed are those of the authors and do not reflect official positions of the US Department of Commerce.

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1 Introduction

Many important questions in international economics involve the complex activities of multinational enterprises (MNEs) in time and space. Consider the recent rise in protectionism worldwide: the debate on the United Kingdom abandoning the European Union (EU), "Brexit", is one example. Under Brexit, would MNEs pull out from the United Kingdom? Would MNEs located in the EU be affected? How would different implementations of Brexit affect MNEs' expansion patterns? And how would long-run consequences differ from short-run adjustments? Providing sound answers to these and other similar questions requires an understanding of the dynamic patterns of MNE expansion and of the nature of the costs these firms face.

Despite their importance, the behavior of MNEs and their affiliates in time *and* space has received little attention in the literature.[1] On the empirical side, this is primarily due to data limitations. On the theoretical side, the nature of the costs of MNE activities—whether variable, fixed, or sunk, and whether host- or destination-country specific—poses challenges to tractability, particularly in a multi-country dynamic setting where MNEs can separate the locations of production and sales. This paper contributes to filling the gap in the literature by introducing a new quantitative dynamic multi-country model of the MNE, which is informed by a new set of facts on the behavior of foreign affiliates of US MNEs. The model is aimed at answering counterfactual questions about the effects of policy shocks on MNE expansion, which require both a rich spatial and dynamic structure.

Our analysis uses a long panel of US MNEs and their foreign affiliates from the Bureau of Economic Analysis (BEA). We start by documenting two facts about the dynamic behavior of MNEs and their affiliates.[2] First, MNE expansion happens mainly at the extensive, rather than intensive, margin. We observe that MNEs expand by entering new markets, either with a new affiliate or exporting from an existing one. We do not find evidence of growth at the intensive margin within a country: the ratio of affiliate-to-parent sales is flat over the affiliate's life. Second, the vast majority of affiliates are born with sales to the host market, which remains the main destination market of the affiliate; sales to other markets may start later in the affiliate's life.

Guided by these facts, we build a multi-country dynamic model of MNE expansion. Home-based firms decide whether, when, and where to open foreign affiliates. Affiliates, in turn, can sell both to their host market and to any other market. Affiliate operations, both in the host and in the export

[1] See Antrás and Yeaple (2014) for a detailed survey on the main facts and theories about MNEs.

[2] Studying the behavior of US MNEs and their foreign affiliates is a relevant setup not only because the United States are the main source of MNEs in the world, but also because MNE affiliates are the main channel through which US firms reach foreign consumers. In 2009, for instance, majority-owned affiliates of US MNEs abroad accounted for 75 percent of US sales to foreign customers; forty percent of those affiliates' sales were exports, i.e., sales to customers outside the affiliate's host market (Yeaple, 2013).

markets, are subject to sunk, fixed, and variable costs. The MNE decisions of whether to set up an affiliate in a market, and whether to export from it, are shaped by the interaction of firm-specific characteristics, persistent aggregate productivity and demand shocks, and the array of MNE costs.

While the static components of our model are standard and follow Melitz (2003), the way we formulate the dynamic problem of the MNE is new to the international trade literature. We build on insights from the literature on real options to solve general models of investment under uncertainty (see Dixit and Pindyck, 1994). More precisely, our model is based on a *compound option* structure: opening an affiliate in a country is an option, which, if exercised, gives access to a set of additional options, such as exporting from the affiliate to any other location. Guided by the observation that almost all affiliates in the data have some horizontal sales at birth, we assume that firms that decide to do Foreign Direct Investment (FDI) must first set up an affiliate and sell to the local market; only then can they consider exporting from that affiliate.

The compound option structure, coupled with the assumption of sequential MNE activities and a standard Armington assumption, are key for achieving tractability of the model while, at the same time, preserving the rich heterogeneity necessary for quantitative analysis. In particular, it is through this compound option structure that we are able to tractably introduce interdependence in the location choices of the MNE: the decision to open an affiliate in a country depends on the set of countries where the affiliate can export to. Moreover, because of the continuous time specification, the value functions can be solved in closed form as simple additive functions of the firm's realized profit flow, the option value of expansion, and the option value of exit. We further leverage the tractability of the firm's problem by aggregating firms' outcomes to solve for the evolution of the price index in each country. Based on these price indexes, we can construct measures of welfare changes induced by changes in MNEs' activities.

We calibrate the model to static and dynamic moments related to the behavior of US MNE affiliates located in the top ten host countries for US FDI, over thirty years. Our calibration implies that opening and operating affiliates is more costly than exporting from them, for most host countries. Exports to the United States (the Home country) are generally associated with lower barriers than exports to other destinations. Heterogeneity, however, is large across host countries, sales type, and type of frictions.

The calibrated model is also able to reproduce non-targeted observations related to the selection patterns of MNE affiliates within and across host markets, as well as sorting patterns of MNE entry across host markets, in the spirit of the facts documented in Eaton et al. (2011) for French exporters. Concretely, the model matches fairly well the size advantage, in terms of horizontal sales, observed for: affiliates that export over affiliates that do not export; affiliates that export earlier in life over affiliates that export later; and affiliates that are opened first within the MNE over subsequent affiliates. Additionally, the model captures the positive correlation observed between the size of the MNE in its Home market (the United States) and the number of markets penetrated, as well as the negative correlation between the extensive margin of market penetration and market popularity.

Armed with the calibrated model, we perform various counterfactual exercises with the goal of evaluating the effect of changes in different frictions on the dynamics of MNE expansion, and the endogenous responses of price indexes. We also illustrate how the possibility of affiliate exports—the compound option structure—interacts with changes in frictions to deliver very different long-run responses of MNE activity compared to what a dynamic standard model of only horizontal FDI predicts. Since our sample includes affiliates located in the United Kingdom, Ireland, Germany, and France, we use the potential withdrawal of the United Kingdom from the European Union (EU), Brexit, as our main counterfactual exercise. Our counterfactual exercises point to the importance of considering both the time *and* space dimensions when evaluating the changes in the MNE expansion decisions after a shock.

Potential implementations of Brexit are related to the increase of different types of export costs between the United Kingdom and other EU countries. Our model predicts that an increase in export costs between the United Kingdom and other EU countries would have a static effect, a dynamic effect, and an equilibrium price effect. First, export activities between the United Kingdom and the EU would become more costly, so that sales from UK-based affiliates to the EU, as well as sales from EU-based affiliates to the United Kingdom, would decline. This decline would drive a decrease in the incentive to open affiliates in the United Kingdom and in other EU countries, due to the smaller, and costlier, export market. Second, increases in trade costs would affect the affiliate export band of inaction, and hence, affiliate export entry and exit rates would change. This second effect is driven by including one-time sunk costs that are distinct from fixed per-period costs–a feature only possible to include in dynamic models. Finally, increases in trade frictions would have the effect of raising prices not only in the United Kingdom, but also in the EU, encouraging more export entry from the United Kingdom into those markets. The strength of each effect on aggregate firm dynamics varies depending on the nature of the shock to export costs. For instance, while increasing sunk export costs would increase both the sales and number of affiliates selling to the EU from the United Kingdom, increases in per-period fixed costs would decrease both the number and sales of UK-based US affiliates to the EU. Again, these different patterns could not be captured with a static model of the MNE since one-time sunk and fixed per-period costs would be indistinguishable.

This paper is related to the existing literature in several ways. First, most contributions in the literature have analyzed MNE behavior in space, but not in time. Papers such as Ramondo and Rodríguez-Clare (2013), Tintelnot (2017), Fan (2017), Arkolakis et al. (2018), Alviarez (2019), and Head and Mayer (2019) have made substantial progress in building static quantitative models that allow MNEs to set up affiliates in countries that differ from the destinations of their sales.[3] Making progress in dynamic setups, while keeping the spatial complexity of the static models, requires restricting the problem of the MNE to retain tractability. The sharp patterns that we document from observing MNE affiliates over time guide us on how to simplify this problem: thanks to our novel compound option structure, coupled with the assumption on the sequentiality of MNE decisions, we are able to reduce the choice set of firms in a way that is consistent with the data, while keeping the model amenable to quantitative analysis. In this way, we make substantial progress towards modeling the dynamics of MNE expansion, without sacrificing the spatial richness of static models.

Second, there is a small, but growing, literature that analyzes different aspects of the dynamic behavior of the MNE. Papers in this literature, however, limit the spatial dimension of the problem. Gumpert et al. (2018) focus on the life-cycle dynamics of exporters and MNEs as alternative ways of serving a foreign market, and assess the role of MNEs on new exporters' dynamics. Given the nature of their question, the analysis does not consider export platforms, and focuses on life-cycle, rather than aggregate, firm dynamics. Fillat and Garetto (2015) build a dynamic two-country model of exporters and MNEs, where they introduce the idea that MNE activities can be treated as a real option that gets exercised once an affiliate is opened abroad.[4] Fillat et al. (2015) extend this idea to a multi-country setup. Both papers focus on the link between the MNE expansion decisions and asset prices, and both assume that the activities of affiliates are restricted to their market of operation. Our model treats MNE activities as a compound, rather than a simple, option. In this way, we are able to preserve the tractability of the problem in a dynamic multi-country setup, and expand on the spatial dimension by separating the location of production and sales. [5]

Third, our paper is naturally related to the large literature on export dynamics, which has been primarily concerned with quantifying the various costs of export activities and their welfare implications.[6] An important difference of our approach from the literature on export dynamics is that

[3] In static setups, the seminal model in Helpman et al. (2004) and the quantitative version of it in Irarrazabal et al. (2013) assume that both the location of production and sales of the MNE coincides—that is, MNE activities are restricted to horizontal sales, which are the alternative to exports for serving foreign consumers.

[4] Impullitti et al. (2013) also use the real option analogy to model the entry and exit patterns of exporters.

[5] Other papers in the MNE literature limit both the spatial and dynamic dimension of the analysis by considering only horizontal FDI sales and only two periods (see, for instance, Ramondo et al., 2013; Egger et al., 2014; Conconi et al., 2016).

[6] Earlier contributions by Baldwin and Krugman (1989), Roberts and Tybout (1997), Das et al. (2007), and

the nature of the MNE problem is more complex than the exporter problem: MNEs choose not only which markets to serve, as an exporter does, but also the location from which to serve each of those markets. Our compound option structure allows us to solve the complex spatial problem of the MNE in a multi-country dynamic setup. In this way, we complement the literature on export dynamics by quantifying the frictions to MNE expansion, and by analyzing their implications in terms of aggregate firm dynamics and welfare.

Finally, our paper relates to the large literature that analyzes the dynamics of domestic firms, which goes back to Davis et al. (1996), and more recently Decker et al. (2014, 2016). Our facts suggest that the dynamics of MNE affiliates are starkly different from the dynamics of domestic firms. We interpret these differences as indicative of the fact that new US firms face a very different set of frictions in the domestic and foreign markets.

2 Evidence on US MNE Expansion

We document two novel facts on the dynamic behavior of foreign affiliates of US multinational enterprises (MNEs): MNE expansion happens at the extensive, rather than the intensive, margin; and the vast majority of affiliates are born specialized in sales to the host market, which remains the main activity of the affiliate, while export activities may start later in life.

2.1 Data

Our empirical analysis uses firm-level data on the operations of US MNEs from the Bureau of Economic Analysis (BEA). The data include detailed information on the operations of MNEs in the United States and their affiliates abroad, for the period 1987-2011. We restrict the sample to majority-owned affiliates that do not operate in tax haven countries, have manufacturing as their primary activity, and belong to a US parent operating in any sector.[7] We further consolidate affiliates belonging to the same parent and operating in the same country and 3-digit industry.

Alessandria and Choi (2007) find evidence of large sunk costs of exporting by focusing on observed patterns of export entry and exit. Subsequent analyses, such as Eaton et al. (2008) and Ruhl and Willis (2017), incorporate facts related to the life-cycle dynamics of new exporters and find that those costs are much lower. Alessandria et al. (2018) take a further step and also calculate the welfare gains from trade in a dynamic setting that matches well the life-cycle export facts. Arkolakis (2016) presents rich micro evidence on firm selection and export growth that supports dynamic theories of endogenous entry costs *vis-á-vis* standard export sunk costs. Finally, Fitzgerald et al. (2017), using detailed data on export prices and quantities, show that the life-cycle growth patterns of those two variables are quite different.

[7] Our sample is primarily composed of affiliates that are majority owned during their whole life. Only about one percent of affiliates go from majority to minority owned and less than two percent go from minority to majority owned.

Table 1: Number of observations, by sale type.

	Horizontal sales	Export sales
No. of observations	38,080	38,080
with positive sales	36,135 (95%)	25,958 (68%)
of pure type	14,035 (37%)	2,418 (6.3%)
Sales accounted by pure type	15.6%	7.7%

Note: Observations are at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. A pure-type affiliate is an affiliate for which at least 99 percent of sales are either only horizontal or only export sales.

Finally, for the facts presented in this section, we focus on affiliates that open during our sample period and that survive for at least ten consecutive years in the market. This restriction implies that we exclude affiliates that open in 2003 or later, as well as observations belonging to the affiliate's eleventh year of life, or greater. We also remove affiliates and parents with zero total sales.[8]

Crucially, the BEA data break down affiliate sales by destination: the host market of operation (horizontal sales), and other markets (exports). The data further distinguish between affiliate exports to the United States and to third markets. Every five years the BEA conducts a more detailed benchmark survey, which further distinguishes affiliate exports to Canada, the United Kingdom, and Japan.[9] However, the BEA data do not record total exports from the US parent by destination.

Table 1 shows the number of observations with positive horizontal and export sales in our sample. Almost 95 percent of our affiliate-year observations have some horizontal sales, while more than two-thirds of them have some exports. More than one-third of the observations correspond to affiliates with horizontal sales only, while the share of affiliates with only exports is around six percent. Since affiliates that only export are few and account for a small share of total affiliate sales, the model we present in Section 3 does not feature pure exporters.

Appendix A provides more details on the data coverage and sample construction.

[8] This restricted sample covers 23 percent of all affiliates in manufacturing as well as 38 percent of their total sales. Facts computed using a larger sample with a five-year survival threshold display the same patterns (not shown).

[9] The distinction between the United States and other export markets of the affiliate does not make any substantial difference for the facts documented below. We do use, however, the available break-down of affiliate exports by destination in our calibration.

2.2 The expansion of MNE affiliates

We start by documenting that MNE affiliates grow by entering new export markets.

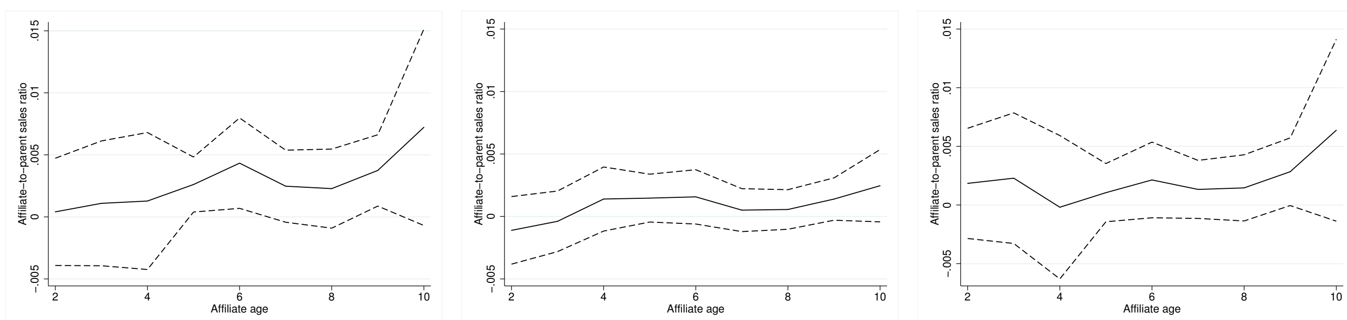
Figure 1 shows the ratio of affiliate-to-parent sales, by affiliate age, for all, horizontal, and export sales. We plot the coefficients on age dummies from estimating, by Ordinary Least Squares (OLS),

$$\log Y_{iap} = \sum_{a=2}^{10} D_{iap} + \beta \log \text{global emp}_{iap} + u_{iap}. \tag{1}$$

The dependent variable Y_{iap} denotes the ratio of affiliate-to-parent sales for affiliate i belonging to parent p at age a, while D_{iap} is a set of age dummies. We include country-year and affiliate fixed effects in all the specifications as well as a control for the global size of the corporation in terms of employment, $\log \text{global emp}_{iap}$.

Figure 1: Affiliate-to-parent sales ratio, intensive margin.

(a) All sales (b) Horizontal sales (c) Export sales



Notes: OLS coefficients on age dummies (relative to the entry year) from estimating (1), with affiliate fixed effects and country-year fixed effects. The dependent variable *affiliate-to-parent sales ratio* refers to affiliate sales relative to the domestic sales of the US parent. Five-percent confidence intervals shown in dashed lines. Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Each panel includes affiliates with positive sales in the given category.

The affiliate-to-parent sales ratio is flat: at all ages, this ratio is not significantly different from the ratio in the entry year. A similar lack of growth is observed not only for all affiliate sales, but also for horizontal and export sales, separately.[10]

MNE affiliates do not grow at the intensive margin, but they do expand at the extensive margin, i.e., adding destinations other than the host market. Table 2 shows the results from estimating, by

[10] Notice that this finding is in stark contrast with the export dynamics literature, which documents small export shares at entry and intensive margin growth in exports over the life of surviving exporters. See, among others, Arkolakis (2016), Fitzgerald et al. (2017), and Ruhl and Willis (2017).

OLS,

$$\log Y_{iap} = \sum_{\delta \in \{-5,...1\} \cup \{1,...5\}} D_{i\delta p} + \beta \log \text{global emp}_{iap} + u_{iap}, \tag{2}$$

where $D_{i\delta p}$ equals one when affiliate i is δ years away from starting exporting. We also control for the global employment of the MNE, and include country-year and affiliate fixed effects.

Results in Table 2 are relative to sales in the year of entry into exports, the excluded category. In each of the five years preceding export entry (-5- to -1), the ratio of affiliate-to-parent sales is significantly lower than the ratio at entry. After export entry (1 to 5), this ratio is flat. The similarity of the coefficients for $\delta = \{-5, \ldots, -1\}$ indicates that the affiliate-to-parent sales ratio increases only at the time of export entry. Expansion happens only at the extensive margin of sales destinations.[11] Notice that the flat affiliate sales profile observed in Figure 1a is not inconsistent with the jump observed in affiliate sales at export entry documented in Table 2 because affiliates start exporting at different ages.

Robustness. One could argue that the lack of growth in MNE affiliates' sales may be due to the fact that the affiliate "inherits" the age of the parent so that, *de facto*, it is a much older firm, and hence, has lower growth rates. This may well be happening, as documented for multi- versus single-plant firms in the United States by Kueng et al. (2017).[12] Unfortunately, the BEA data do not record the age of the parent firm. However, we can look at the affiliate position in the opening sequence of the MNE—i.e., first affiliates versus subsequent affiliates. In this way, we can compare affiliates belonging to younger MNEs with affiliates belonging to older MNEs (or the same MNE at older ages). Columns 1 and 2 in Appendix Table B.1 show that first affiliates do not appear to grow faster than subsequent affiliates—even though the age-dummy coefficients for subsequent affiliates are more precisely estimated.

A second argument can be that, since global value chains (GVCs) have been growing very fast in the last decades, affiliates linked to them might be growing faster than non-GVC affiliates. In columns 3 and 4 in Appendix Table B.1, we show the results of estimating (1) separately for GVC affiliates (defined as affiliates with positive intra-firm exports), and for non-GVC affiliates (defined as affiliates with zero intra-firm exports). Both groups of affiliates have flat affiliate-to-parent sales ratios—even though estimates are more precise among GVC affiliates.

[11] It is worth noting that MNE expansion does not happen by opening more than one affiliate in a given host market. Only 6.98 percent of US MNEs have more than one affiliate in the same host country; at the country-MNE level the share is 4.6 percent; and at the country-firm-year level the share is only 3.38 percent.

[12] Kueng et al. (2017) document a stark difference in the life-cycle employment profiles of establishments belonging to single- versus multi-unit firms in manufacturing: while establishments in single-unit firms grow steeply, the ones in multi-unit firms do not grow.

Table 2: Affiliate-to-parent sales ratio, extensive margin.

D(years to export entry = -5)	0.001
	(0.010)
D(years to export entry = -4)	-0.023**
	(0.010)
D(years to export entry = -3)	-0.024*
	(0.013)
D(years to export entry = -2)	-0.018**
	(0.009)
D(years to export entry = -1)	-0.019***
	(0.007)
D(years to export entry = 1)	-0.018*
	(0.009)
D(years to export entry = 2)	-0.014
	(0.009)
D(years to export entry = 3)	-0.005
	(0.010)
D(years to export entry = 4)	-0.003
	(0.009)
D(years to export entry = 5)	-0.005
	(0.008)
log global employment	-0.012
	(0.009)
Obs	38,080
R^2	0.011

Note: Results from estimating (2) by OLS. Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. The dependent variable *affiliate-to-parent sales ratio* refers to affiliate sales relative to the domestic sales of the US parent. All specifications include affiliate and country-year fixed effects. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

A third argument for the observed lack of growth in MNE affiliates' sales is related to the mode of FDI entry. If MNEs establish foreign affiliates mostly through a merger with—or an acquisition of—an existing firm (M&A), one could argue that "new" foreign affiliates are in reality preexisting firms that likely grew before their acquisition. The BEA asks a subset of affiliates whether they were created through an M&A or a greenfield project.[13] Columns 5 and 6 in Appendix Table B.1 show that, relative to age two (the first year for which sales are recorded for the whole year), the sales ratio grows very little, regardless of the mode of FDI entry.

A final concern is that the flat sales ratio observed for affiliates may be due to the fact that firms grow in a foreign market first through exports, and only subsequently through opening an affiliate. Since the BEA data do not include information about parent exports by destination market, we are not able to address this question directly. Gumpert et al. (2018), however, report that, for Norway and France, the difference in growth profiles for MNEs with previous export experience into a market and those without it is not significant, except for the first year of the affiliate's life.

The fact presented in this section motivates an important feature of our model: MNE affiliates grow by entering new destination countries.

2.3 The activities of MNE affiliates over time

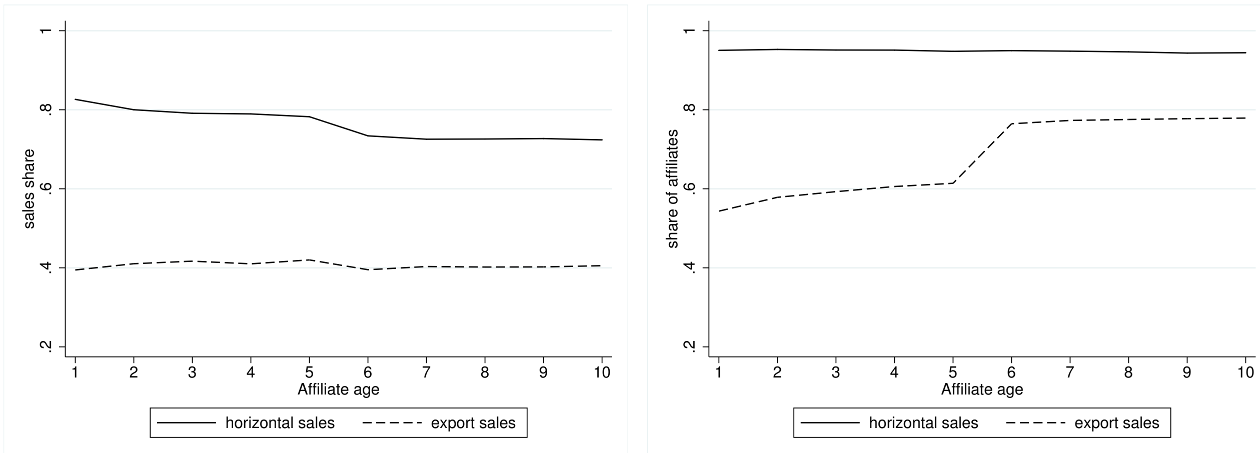
We now present evidence on the specialization patterns of affiliates in terms of horizontal and export activities over time. We show that affiliates are born specialized in horizontal sales, and they may incorporate exports as a secondary activity later in life.

Figure 2 shows the evolution of the intensive and extensive margins of horizontal and export sales as shares of total sales of the affiliate. Figure 2a shows the evolution of the horizontal and export sales share, computed as an average over affiliates reporting, respectively, positive horizontal and positive export sales. On average, horizontal sales account for about 80 percent of affiliate sales at birth and decrease by ten percentage points over the first ten years of life of the affiliate, while the export share is flat at 40 percent. To capture the extensive margin of horizontal and export sales, Figure 2b plots the percentage of affiliates with non-zero horizontal sales and non-zero export sales. While the share of affiliates with horizontal sales is stable at more than 95 percent, the share of exporting affiliates increases from 50 to 70 percent during the first ten years of life of the affiliate. In other words, for horizontal activities, changes in sales shares are due to the intensive margin, while export shares increase only because of affiliates that start exporting. Hence, the data suggest

[13] This question applies only to firms that opened mid-year, and thus, the reported information about sales covers only part of the entry year. For this reason, we compute the ratio of affiliate-to-parent sales starting at age two.

Figure 2: Intensive and extensive margins of affiliate sales, by type.

(a) Affiliate sales shares (b) Share of affiliates



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Horizontal and export sales refer, respectively, to sales to the market where the affiliate is located, and to sales to markets outside the local market. (2a): average sales, as a share of total affiliate sales, include affiliates with positive horizontal and export sales, respectively. (2b): number of affiliates, as a share of the total number of affiliates, include affiliates with positive horizontal and export sales, respectively.

that, over time, affiliates incorporate export sales into their activities, but they never stop selling to their host market.

The patterns in Figure 2 are confirmed by OLS regressions that include a battery of fixed effects, as shown in Appendix Table B.2. Estimates that include affiliate fixed effects suggest that, on average, horizontal (export) sales shares decrease (increase) during the life of an affiliate, and the share of affiliates with exports is higher among older affiliates.

Robustness. Appendix Figure B.1 and Table B.3 report analogous results for the subset of affiliates that are pure type at birth—i.e., firms that in their first year of life either sell exclusively to the host market or only export. The results on pure-type affiliates reinforce the patterns shown in Figure 2: pure-type affiliates diversify activities over their life, moving from exclusively horizontal sales to also exporting. Relatedly, Appendix Figure B.2 shows the relationship between the intensity at which an activity is performed and the time at which the affiliate first starts that activity: the older the affiliate is when it starts exporting, the lower its export intensity.

The fact documented in Figure 2 motivates a key assumption of our model: all affiliates start operations with some horizontal sales and may endogenously expand into export markets.

3 A Dynamic Model of MNE Expansion

We build a quantitative dynamic model in which MNEs open affiliates across countries over time. Affiliates sell in their host markets, and they choose whether to export to other markets from there. We impose assumptions that are guided by the facts documented in Section 2 and are key for the tractability of the model.

The main innovation of the model is the introduction of a compound option formulation that allows us to characterize the richness of the decisions of the MNE in time and space. This formulation is novel to the international trade literature, and it is the key element that makes the model amenable to quantitative analysis.

3.1 Preferences and technology

The economy consists of $N+1$ countries: the Home country (the United States in our data) and N foreign countries. Time is continuous. In each country k, consumers have preferences over a composite good,

$$U_k = \int_0^\infty e^{-\rho t} Q_k(t) dt, \tag{3}$$

with ρ denoting the subjective time discount rate. The quantity $Q_k(t)$ aggregates a continuum of varieties, indexed by v, with a constant elasticity of substitution (CES) $\eta > 1$,

$$Q_k(t) \quad = \quad \left[\sum_i \sum_j \int_{\Omega_{ijk}(t)} \lambda_{ij}^{\frac{1}{\eta}} q_{ijk}(v,t)^{\frac{\eta-1}{\eta}} dv \right]^{\frac{\eta}{\eta-1}}. \tag{4}$$

The term $q_{ijk}(v,t)$ denotes consumption of variety $v \in \Omega_{ijk}(t)$, and $\Omega_{ijk}(t)$ denotes the set of varieties sold to country k and produced by affiliates located in j belonging to firms from i, at time t. The term λ_{ij} denotes a preference shifter.

Assumption 1 (Armington). *Varieties consumed and produced are firm-location specific.*

As in Armington (1969), Assumption 1 states that consumers perceive differently varieties produced in different locations by the same firm, a standard assumption in the literature. For example, consumers in a given destination perceive Möet Chandon champagne produced in France as different from Chandon sparkling wine produced by the same firm in the United States.

Each country is populated by a continuum of firms. The Home country is the only source of MNEs: Home firms decide whether to operate only in their home market or to also establish affiliates abroad. For this reason, to simplify notation, we remove the index i that denotes a variety's origin country and use the subscript d to refer to the parent's operations at Home.

Labor is the only factor of production. Each firm produces with a linear technology and operates under monopolistic competition. As in Melitz (2003), each firm is characterized by a productivity parameter φ that determines the unit labor cost of the good produced. Each firm sets prices to maximize profits from sales to each destination, $p_{jk}(\varphi) = \widetilde{\eta} c_{jk}(\varphi)$, with $\widetilde{\eta} \equiv \eta/(\eta-1)$ and $c_{jk}(\varphi) \equiv w_j \tau_{jk}/\varphi$. The term w_j denotes the wage in country j where production takes place, and τ_{jk} denotes the iceberg cost of shipping goods from production location j to destination k, with $\tau_{jk} \geq 1$, $\forall j \neq k$, and $\tau_{jj} = 1$, $\forall j$.

A Home firm's domestic profits are given by $\pi_d(\varphi) = H(w_d/\varphi)^{1-\eta} P_d^{\eta} \lambda_d Q_d$, while variable profits from sales to k of an affiliate in j are given by $\pi_{jk}(\varphi) = H(\tau_{jk} w_j/\varphi)^{1-\eta} \lambda_j P_k^{\eta} Q_k$, where $H \equiv \eta^{-\eta}(\eta-1)^{\eta-1}$ and P_k is the corresponding CES price index. Note that, for $j = k$, the variable π_{jk} denotes profits from horizontal sales, while for $j \neq k$, it denotes profits from affiliate export sales.

When a firm establishes an affiliate in a foreign country j, it has to pay a sunk entry cost $F_j^h > 0$. The affiliate starts by selling locally and incurs a per-period fixed cost $f_j^h > 0$. Once the affiliate is in place, it can expand its operations to export to other markets. An affiliate located in country j has to pay a sunk cost $F_{jk}^e > 0$ to start exporting to country k, and a per-period fixed export cost $f_{jk}^e > 0$. For simplicity, we assume that there are no per-period fixed costs associated with domestic production, so that all firms produce at Home.[14]

Our setup has two important implications. First, Assumption 1, coupled with CES preferences and monopolistic competition, implies that there is no cannibalization of sales when a MNE serves a market by opening an affiliate there and by exporting to it from an affiliate in a different location; those affiliate exports and horizontal sales refer to different goods. Second, Assumption 1 also implies that the affiliate entry decision of the MNE is separable across locations: affiliate profits in one location do not depend on the number of affiliates of the same MNE in other locations.[15]

Our data on US MNEs offer evidence in support of the implications of our setup regarding the lack of cannibalization and separability in the affiliate-entry decisions. First, a large share of firms have

[14] The model can easily accommodate exports from the Home country: this is the case of $j = i$. However, in our calibration, we do not include exports from the US parent because the BEA data do not break down the parent exports by destination (and more generally, we observe very limited information on the activity of the parent in the United States). The model with exports will deliver aggregate predictions that are in line with the predictions of the proximity-concentration tradeoff in Brainard (1997): the higher trade costs and the lower plant-level scale economies, the larger is the share of foreign sales accounted for by affiliate sales relative to export sales.

[15] Assumption 1 makes the product space in our model different from Tintelnot (2017). In his setup, varieties are not unique to a firm-location pair. As a result, because of the presence of fixed production costs, firms solve a complex static combinatorial problem to choose the minimum cost location where to produce a variety for a given destination market. The solution of such a problem would be unfeasible in a dynamic multi-country setting.

affiliate exports to a country in which they also operate affiliates. For the benchmark year 2004, we are able to examine this pattern for three countries (Canada, the United Kingdom, and Japan). Of the 20,359 (5,017; 5,224) affiliates that export to Canada (United Kingdom; Japan), 64 (70; 47) percent belong to a US parent that also has affiliates located in Canada (United Kingdom; Japan). Second, Appendix Figure B.3 shows that the horizontal sales of an affiliate in a country do not decrease when another affiliate of the same parent starts exporting to that country from another location.[16] Finally, the comparison between conditional and unconditional probabilities of affiliate entry into a country informs us about the separability of the MNE entry decisions. Appendix Table B.5 shows that, for a given US parent, there is an extremely small –and in several cases statistically insignificant– difference between the unconditional probability of opening an affiliate in a country and the probability of opening an affiliate conditional on already having an affiliate in a country with similar characteristics (i.e., located in the same continent, with common border, with common language, or with similar income per capita).[17]

3.2 The MNE dynamic problem: the compound option

We now present the MNE dynamic problem. At each point in time, a firm endogenously decides whether to open an affiliate in a foreign country, and whether—and where—to export from its existing affiliates, including exporting to the Home market. A firm may also decide to shut down affiliates, or to exit any of its affiliate export markets.

We use the notion of a *compound option* to model the dynamic problem of the MNE. Opening an affiliate in a country is an option that, when exercised, gives access to another set of options, namely the possibility of expanding to each export destination. Hence, the decision to open an affiliate in country j depends on the set of countries where the affiliate can export to. The compound option structure allows us to easily solve the firm's problem backwards, as suggested by Dixit and Pindyck (1994, chap. 10). Conditional on the MNE having an affiliate in country j, one can solve for the value of exports to each destination and for the policy functions that induce the affiliate to start, or stop, exporting to each country $k \neq j$. Together with the value of horizontal sales, the value of

[16] We restrict the sample to three destinations for which we can observe arm's length affiliate exports: Canada, the United Kingdom, and Japan. The results are robust to including controls for the global operations of the parent and the operations in the United States, a battery of fixed effects, different clustering of standard errors, and to considering different samples of affiliates as well as of exports (arm's length and intra-firm).

[17] This finding is in stark contrast with the findings for exporter entry in Morales et al. (2018): they find, for instance, that the unconditional probability of exporting to a given country is 0.7 percent and increases to 6.7 percent if the firm is already exporting to an adjacent country. In contrast, we find that the unconditional probability of opening an affiliate in the United Kingdom is 2.5 percent and increases to only three percent if the MNE already has an affiliate in an adjacent country. In general, while differences between conditional and unconditional probabilities for exporter entry range between 2 and 4 *times*, differences for MNE entry range between 2 and 20 *percent*. This finding is robust to considering different samples of MNEs and affiliates (not shown).

exports determines the value of an affiliate in country j. One can then solve for the policy functions that induce the firm to open, or shut down, the affiliate.

The assumption we present next is guided by the empirical observations in Section 2.

Assumption 2 (**Sequential MNE activities**). *A new affiliate has to sell to its host market before eventually starting to export from there.*

Assuming sequential decisions for the affiliate is a mere artifact to gain tractability: because the model is specified in continuous time, opening an affiliate and exporting from it can happen almost simultaneously. In this way, the model can generate affiliates that export from birth, as observed in the data.

Following Ghironi and Melitz (2005), we define the firm-level productivity φ as the product of a time-invariant firm-specific component, z, and a time-varying Home-country specific component, Z, so that $\varphi \equiv z \cdot Z$. The term z is firm-specific, drawn from a time-invariant distribution, $G(z)$, as in Melitz (2003). We assume that $Z = e^X$, where X is a Brownian motion with drift,

$$dX = \mu dt + \sigma dW, \tag{5}$$

for $\mu \in \Re$, $\sigma > 0$, and dW denoting a standard Wiener process. Additionally, we introduce host-country aggregate demand shocks by assuming that aggregate demand in destination country k evolves according to a geometric Brownian motion,

$$dQ_k = \mu_k Q_k dt + \sigma_k Q_k dW_k, \tag{6}$$

where $\mu_k \in \Re$, $\sigma_k > 0$, and dW_k denotes a standard Wiener process, possibly correlated with the Home aggregate productivity shock.[18] We assume that when a firm operates an affiliate in a foreign country, it transfers both the aggregate and the idiosyncratic components of the productivity shock to the host market. In this way, MNE operations contribute to the transmission of productivity shocks across countries, in the spirit of Cravino and Levchenko (2017).

Our shock structure is based on analytical and computational convenience, as well as on empirical observations. Analytically, the specification in (5)-(6) is equivalent to assuming that aggregate Home productivity and foreign demand growth behave like a random walk and that productivity and demand growth are independently and identically distributed. This is a convenient functional form assumption that guarantees the tractability of the model's solution. More precisely, affiliate

[18]The shock process (6) is analogous to assuming that foreign productivity X_k evolves according to a process that, in equilibrium, implies that foreign demand evolves according to (6).

profits from sales to country k are linear in the term $e^{(\eta-1)X}Q_k$. Thus, it is convenient to define the "composite" shock $Y_k \equiv e^{(\eta-1)X}Q_k$, which captures the effect of both source- and destination-country aggregate shocks on affiliates' profits. The composite shock Y_k is also a geometric Brownian motion with drift $\tilde{\mu}_k$ and variance $\tilde{\sigma}_k^2$ given by

$$\tilde{\mu}_k = \mu_k + \mu(\eta-1) + \frac{\sigma^2}{2}(\eta-1)^2 + \gamma_k\sigma_k\sigma, \tag{7}$$

$$\tilde{\sigma}_k^2 = \sigma_k^2 + \sigma^2(\eta-1)^2 + 2(\eta-1)\gamma_k\sigma_k\sigma, \tag{8}$$

where γ_k denotes the correlation between e^X and Q_k. We show below that the model can be solved in terms of realizations of the composite shock. Computationally, relying only on aggregate shocks makes feasible the aggregation of individual firms' decisions and the computation of equilibrium price indexes for many countries. By relying on aggregate shocks only, we do not need to keep track of changes in the firms' productivity distribution over time, which significantly reduces the dimensionality of the state space. Furthermore, the introduction of country-specific demand shocks allows the model to match the evolution of affiliate sales shares in different host countries.

Empirically, this specification is broadly consistent with the main sources of variation observed in the data. Most of the variation in US MNE affiliates' sales is explained by country-specific time-varying shocks and parent fixed effects, rather than parent- and affiliate-level time-varying shocks (see Appendix Table B.4). Finally, our shock structure is consistent with the facts shown in Section 2. Country-level shocks, together with the assumption that the MNE transfers its productivity to its affiliates abroad, imply that, conditional on entry, a firm's Home and foreign sales perfectly co-move, as observed in the data (see Figure 1). The persistence of the aggregate shock, together with aggregate productivity growing over time ($\mu \geq 0$), gives rise to the dynamic patterns documented in Figure 2: affiliates start serving their host market, and later on, they start expanding internationally.

Bellman equations. The state of the economy is described by the $(N+1)$-tuple $(X, \mathbf{Q})$, where $\mathbf{Q} = [Q_1, \ldots, Q_N]$. Let $\mathcal{V}(z, X, \mathbf{Q})$ denote the expected net present value of a Home-country firm with productivity z that follows an optimal policy when the state of the economy is $(X, \mathbf{Q})$,

$$\mathcal{V}(z, X, \mathbf{Q}) = V_d(z, X) + \sum_{j=1}^{N} \max\left\{V_j^o(z, X, \mathbf{Q}), V_j^a(z, X, \mathbf{Q})\right\}. \tag{9}$$

The function $V_d(z, X)$ is the value of domestic operations, $V_j^o(z, X, \mathbf{Q})$ is the option value of opening an affiliate in country j, and $V_j^a(z, X, \mathbf{Q})$ is the value of an affiliate in country j, regardless of the destination of its sales. In turn, the value of an affiliate in country j is given by

$$V_j^a(z, X, \mathbf{Q}) = V_j^h(z, X, \mathbf{Q}) + \sum_{k \neq j} \max\left\{V_{jk}^o(z, X, \mathbf{Q}), V_{jk}^e(z, X, \mathbf{Q})\right\}, \tag{10}$$

The function $V_j^h(z, X, \mathbf{Q})$ is the value of horizontal sales in country j, $V_{jk}^o(z, X, \mathbf{Q})$ is the option value of exporting to country k for an affiliate located in j, and $V_{jk}^e(z, X, \mathbf{Q})$ is the value of exports to country k for an affiliate located in j. Equations (9)-(10) reflect Assumption 2. The problem is formulated as a compound option because opening an affiliate in a country is equivalent to exercising an option that gives access to another set of options: the options to export to any other country.

Since all firms operate in the domestic market, the value of domestic operations is simply given by the evolution of domestic profits over time, and depends only on the domestic shock X. Over a generic time interval Δt,

$$V_d(z, X) = \frac{1}{1 + \rho\Delta t}\left[\pi_d(z, X)\Delta t + E[V_d(z, X')|X]\right], \tag{11}$$

where X' denotes the realization of Home aggregate productivity next period.

If a domestic firm has not yet opened an affiliate in country j, all the value from its operations in j is option value—i.e., the value of the possibility of entering j in the future,

$$V_j^o(z, X, \mathbf{Q}) = \max\left\{\frac{1}{1 + \rho\Delta t}E[V_j^o(z, X', \mathbf{Q}')|(X, \mathbf{Q})]; V_j^a(z, X, \mathbf{Q}) - F_j^h\right\}, \tag{12}$$

where $\mathbf{Q}'$ denotes the vector of realizations of demand shocks next period. This equation captures the fact that a firm may keep the option of entering market j, or may enter country j by opening an affiliate there, in which case it pays the entry cost F_j^h and gets the value of having an affiliate in country j, $V_j^a(z, X, \mathbf{Q})$. Because goods are firm- and location-specific, each firm evaluates entering each location separately.

Since we assume that all affiliates sell in the market where they are located, the value of horizontal sales for an affiliate in country j is given by

$$V_j^h(z, X, \mathbf{Q}) = \max\left\{\frac{1}{1 + \rho\Delta t}\left[(\pi_{jj}(z, X, \mathbf{Q}) - f_j^h)\Delta t + E[V_j^h(z, X', \mathbf{Q}')|X, \mathbf{Q}]\right]; V_j^o(z, X, \mathbf{Q})\right\}. \tag{13}$$

This equation captures the fact that the affiliate may survive and make profits from horizontal sales in j, or may shut down, in which case it gets the value of the option of opening an affiliate in j,

$V_j^o(z, X, \mathbf{Q})$.

As indicated by (10), the value of an affiliate is given by the value of its horizontal plus its export sales. The Bellman equation describing the value of the option to export to country k for a firm with an affiliate in country j is given by

$$V_{jk}^o(z, X, \mathbf{Q}) = \max\left\{\frac{1}{1+\rho\Delta t} E[V_{jk}^o(z, X', \mathbf{Q}')|(X, \mathbf{Q})]; V_{jk}^e(z, X, \mathbf{Q}) - F_{jk}^e\right\}. \tag{14}$$

This equation captures the fact that the affiliate may keep the option of exporting to country k—and get the continuation value of that option—or may start exporting to country k, in which case it pays the entry cost F_{jk}^e and gets the value of exporting to k from j, $V_{jk}^e(z, X, \mathbf{Q})$. In turn, this value is given by

$$V_{jk}^e(z, X, \mathbf{Q}) = \max\left\{\frac{1}{1+\rho\Delta t}\left[(\pi_{jk}(z, X, \mathbf{Q}) - f_{jk}^e)\Delta t + E[V_{jk}^e(z, X', \mathbf{Q}')|(X, \mathbf{Q})]\right]; V_{jk}^o(z, X, \mathbf{Q})\right\}. \tag{15}$$

This equation captures the fact that the affiliate may keep exporting to country k—and get the continuation value of that option—or may stop exporting to country k, in which case it gets the value of the option of exporting to k from j, $V_{jk}^o(z, X, \mathbf{Q})$.

Value functions. The problem can be solved backwards by first solving for $V_{jk}^o(z, X, \mathbf{Q})$ and $V_{jk}^e(z, X, \mathbf{Q})$, conditional on the firm having an affiliate in country j. Given the affiliate's location, the value functions only depend on the Home productivity shock and on the demand shock in destination country k. Since these shocks enter the profit functions linearly, we can replace them with the composite shock $Y_k \equiv e^{(\eta-1)X} Q_k$.

Solving for the value of exports conditional on the affiliate's location is a simple case of interlinked options (see Dixit and Pindyck 1994, ch. 7), with solution given by

$$V_{jk}^o(z, Y_k) = B_{jk}^o(z) Y_k^{\beta_k}, \tag{16}$$

$$V_{jk}^e(z, Y_k) = \frac{\pi_{jk}(z, Y_k)}{\rho - \tilde{\mu}_k} - \frac{f_{jk}^e}{\rho} + A_{jk}^e(z) Y_k^{\alpha_k}. \tag{17}$$

The variables $B_{jk}^o(z) > 0$ and $A_{jk}^e(z) > 0$ are firm-specific parameters, while $\alpha_k < 0$ and $\beta_k > 1$ are the roots of $\tilde{\sigma}_k^2 \xi^2/2 + \left(\tilde{\mu}_k - \tilde{\sigma}_k^2/2\right)\xi - \rho = 0$. The term $B_{jk}^o(z) Y_k^{\beta_k}$ in (16) represents the option value of exporting to country k and is increasing in the realization of the composite shock. Similarly, $A_{jk}^e(z) Y_k^{\alpha_k}$ in (17) is the option value of quitting export market k and is decreasing in the realization of the composite shock—i.e., the option of exiting an export market has a larger value in “bad times”. For each country pair (j, k) and for each firm with productivity z, the parameters $B^o_{jk}(z) > 0$, $A^e_{jk}(z) > 0$, and the thresholds for the realizations of the composite shock that induce the affiliate to start and stop exporting—i.e., the policy functions—can be recovered from the appropriate system of value-matching and smooth-pasting conditions.

Following a similar procedure, one can show that the value of horizontal sales, conditional on having an affiliate in country j, is given by the present discounted value of profits from horizontal sales plus the option value of shutting down the affiliate,

$$V^h_j(z, Y_j) = \frac{\pi_{jj}(z, Y_j)}{\rho - \tilde{\mu}_j} - \frac{f^h_j}{\rho} + A^h_j(z) Y_j^{\alpha_j}, \tag{18}$$

where $A^h_j(z) > 0$ is a firm-specific parameter. As a result, the value of an affiliate in country j can be written as

$$\begin{aligned} V^a_j(z, \mathbf{Y}) \;\; = \;\; & A^h_j(z) Y_j^{\alpha_j} + \frac{\pi_{jj}(z, Y_j)}{\rho - \tilde{\mu}_j} - \frac{f^h_j}{\rho} + ... \\ & \sum_{k \in \mathcal{A}_j(z)} \left[\frac{\pi_{jk}(z, Y_k)}{\rho - \tilde{\mu}_k} - \frac{f^e_{jk}}{\rho} + A^e_{jk}(z) Y_k^{\alpha_k} \right] + ... \\ & \sum_{k \notin \mathcal{A}_j(z)} \left[B^o_{jk}(z) Y_k^{\beta_k} \right], \end{aligned} \tag{19}$$

where $\mathcal{A}_j(z)$ is the subset of countries where an affiliate of firm z located in j exports to, and $\mathbf{Y} = [Y_1, \ldots, Y_N]$. Inspecting (19) it is clear that the compound option structure introduces interdependence in the MNE location choices: the value of an affiliate depends on the set of export destinations available from the affiliate’s host country.

It remains to solve for the decision of a firm to set up an affiliate in country j. The option value of opening an affiliate in j is

$$V^o_j(z, Y_j) = B^o_j(z) Y_j^{\beta_j}. \tag{20}$$

Hence, for each host country j and for each firm with productivity z, the parameters $B^o_j(z) > 0$, $A^h_j(z) > 0$, and the thresholds for the realizations of the composite shock that induce the firm to open and shut down an affiliate can be recovered from the appropriate system of value-matching and smooth-pasting conditions.

Lastly, the value of domestic sales is simply given by the present discounted value of profits from domestic sales,

$$V_d(z, X) = \frac{\pi_d(z, X)}{\rho - \hat{\mu}}. \tag{21}$$

Details on the solution of the dynamic problem of the firm are shown in Appendix C.

3.3 Prices indexes and equilibrium wages

Thanks to the tractability of our multi-country model, we are able to solve for the dynamics of the price index in each country. This calculation entails keeping track of the evolution of the mass of affiliates located in each host country j and serving each destination country k. Appendix C reports the expressions for the price indexes for each country j, the law of motion of the mass of MNEs in each country j, and the law of motion of the mass of firms in j that export to a destination k.

The ability to solve for equilibrium price indexes derives from the choices we made about the setup of the model and shock structure. Traditionally, general equilibrium models of trade dynamics feature firm-level shocks but do not feature sunk costs (see, for example, Luttmer, 2007 and Arkolakis, 2016). Existing dynamic models with sunk costs characterize the equilibrium dynamics for a single firm, as in Das et al. (2007) and Morales et al. (2018), or focus on stationary equilibria where aggregate variables do not change over time, as in Alessandria and Choi (2007). These models are usually formulated in discrete time settings where the firm's value function itself needs to be solved numerically. Our continuous time formulation, coupled with unit root shocks, allows us to solve for the value functions in closed form (up to some constants). By including only aggregate shocks, we can easily solve for the price indexes since we do not need to keep track of the evolution of the firm's productivity distribution.

Finally, in each destination country k, equilibrium wages are determined by imposing the market clearing condition,

$$P_k Q_k = w_k L_k + \Pi_k \tag{22}$$

where L_k denotes (exogenous) market size and Π_k denotes the aggregate profits of firms from country k, which include only domestic profits for $k \neq US$, but both domestic and foreign (affiliate) profits for $k = US$.[19]

[19] Because we take Q_k as given, we do not need to specify L_k. Operationally, L_k can either be fixed or backed-up from the labor market aggregation conditions. With our assumptions, L_k will have the same time-varying properties as Q_k. See Appendix C.

3.4 Model's predictions

In this section, we derive some analytical properties of the model regarding the link between firm-level productivity, host market characteristics, and affiliate entry and export thresholds. In order to show these results analytically, we assume that the fixed costs of affiliate operations are "small," so that there is no endogenous exit of affiliates, either from export markets or from their production locations.[20] Under this assumption, the option-value terms $A^e_h(z)$ and $A^e_{jk}(z)$, in (17), (18), and (19), are zero. Hence, we can obtain closed-form solutions for the affiliate entry and export entry thresholds,

$$Y_j^{OH}(z) = \left(\frac{\beta_j}{\beta_j - 1}\right) \cdot \left(\frac{f_j^h + \rho F_j^h}{\rho} - \mathbf{V}_j^E(z, \mathbf{Y}_{-j})\right) \cdot \left(\frac{\rho - \tilde{\mu}_j}{\kappa_{jj}(z)}\right), \tag{23}$$

$$Y_{jk}^{OE}(z) = \left(\frac{\beta_k}{\beta_k - 1}\right) \cdot \left(\frac{f_{jk}^e + \rho F_{jk}^e}{\rho}\right) \cdot \left(\frac{\rho - \tilde{\mu}_k}{\kappa_{jk}(z)}\right). \tag{24}$$

The term $\kappa_{jk}(z) \equiv H(\tau_{jk} w_j / z)^{1-\eta} P_{jk} \lambda_{jk}$ is a firm-specific revenue term, and $\mathbf{V}_j^E(z, \mathbf{Y}_{-j})$ denotes the total value of exporting from an affiliate in j for a firm with productivity z.[21] Details on the derivation of (23) and (24) are in Appendix C.

Proposition 1. *For a given host-destination pair, more productive firms have lower affiliate entry thresholds and lower affiliate export entry thresholds:* $\partial Y_j^{OH}(z)/\partial z \leq 0$ *and* $\partial Y_{jk}^{OE}(z)/\partial z \leq 0$.

Proof. See Appendix C.

Under the assumption that, $\forall k \neq j$, $Y_j^{OH}(z) < Y_{jk}^{OE}(z)$ (the threshold for the shock realization that induces a firm to open an affiliate is lower than the one that induces the affiliate to export), Proposition 1 implies that: 1) affiliates that are exporters from birth have larger horizontal sales than affiliates born with exclusively horizontal sales; and 2) conditional on Home aggregate productivity—or host-country aggregate demand—increasing over time ($\tilde{\mu} \geq 0$), affiliates that start exporting later in life have lower horizontal sales than affiliates that start exporting earlier in life.

The upper panels of Figure 3 illustrate these predictions. The red and blue lines denote, respectively, the threshold for opening an affiliate in j, $Y_j^{OH}(z)$, and the threshold for starting exports from j to k, $Y_{jk}^{OE}(z)$. They are decreasing functions of the firm's productivity z, and hence, they are invertible functions. In Figure 3a, we assume that the realization of the aggregate shock is Y' and

[20] Numerical simulations reveal that the implications described in this section also hold in the general case where fixed costs are large, and hence, exit thresholds are active.

[21] Notice that if $(f_j^h + \rho F_j^h)/\rho - \mathbf{V}_j^E(z, \mathbf{Y}_{-j}) < 0$, then $Y_j^{OH}(z) < 0$. In this case, a firm with productivity z opens an affiliate in j for any realization of Y_j because the value of its potential export network is larger than the cost of opening the affiliate.

that we observe two firms having affiliates in the same host country j. Firm 1 with productivity z_1 has an affiliate in j with only horizontal sales, while firm 2 with productivity z_2 has an affiliate in j that also exports, so that $z_2 \geq z_j^e(Y') \geq z_1$, where $z_j^e(Y')$ denotes the productivity necessary to export when the realization of the composite shock is Y' (*i.e.*, the inverse of $Y_{jk}^{OE}(z)$). Since $z_2 \geq z_1$, the horizontal sales of the affiliate belonging to firm 2 must be larger than the horizontal sales of the affiliate belonging to firm 1. Now, suppose that the realization of the composite shock increases to $Y'' > Y'$. As illustrated in Figure 3b, $z_1 \geq z_j^e(Y'')$ and firm 1 will start exporting from its foreign affiliate in j. Hence, within a host country, affiliates that export earlier in life are more productive and exhibit larger horizontal sales than affiliates that start exporting later.

Proposition 1 also implies that the model exhibits sorting of MNEs across different host markets. First, MNEs with larger parent sales enter more foreign markets by opening foreign affiliates. Second, the mass of firms having affiliates in n host markets is decreasing in n, so that there is a negative relationship between the number of firms having affiliates in n markets and their parent sales (see Corollaries 1 and 2 in Appendix C).

Proposition 2. *For a given firm with productivity z, the affiliate entry threshold is decreasing in the host-market preference shifter, $\partial Y_j^{OH}(z)/\partial \lambda_j \leq 0$, and increasing in the entry cost, $\partial Y_j^{OH}(z)/\partial F_j^h \geq 0$.*

Proof. See Appendix C.

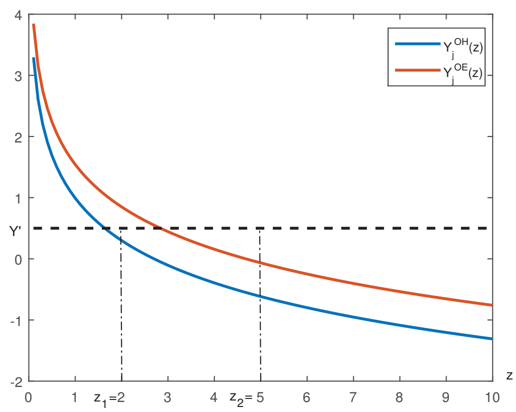
Proposition 2 relates to the expansion strategies of a MNE across countries. Since entry thresholds are decreasing in the preference shifter, the model predicts that—keeping host market size constant and conditional on aggregate productivity or demand increasing over time ($\tilde{\mu} \geq 0$)—a MNE first opens its largest affiliates and subsequently opens its smaller affiliates. Similarly, since entry thresholds are increasing in entry costs, the model predicts that a MNE first opens affiliates in markets that are less costly to enter.

The lower panels of Figure 3 illustrate the predictions of Proposition 2. Figure 3c plots entry thresholds in two host countries of the same size, $(Q_k = Q_j)$ but with different taste shifters $(\lambda_{kk} < \lambda_{jj})$, so that $Y_k^{OH}(z, \lambda_{kk}) \geq Y_j^{OH}(z, \lambda_{jj})$. Firm z only opens an affiliate in country j when the realization of the aggregate shock is Y'. When the realization of the shock grows to $Y'' > Y'$, the firm can also afford to open an affiliate in country k, illustrating that, controlling for factor costs and host country size, a MNE opens its largest affiliates first. Figure 3d plots entry thresholds in two host countries with different entry costs, $F_k^h > F_j^h$, so that $Y_k^{OH}(z, F_k^h) \geq Y_j^{OH}(z, F_j^h)$. Firm z opens an affiliate in country j when the realization of the composite shock is Y'. When the realization of the composite shock increases to $Y'' > Y'$, the firm can also afford to open an affiliate

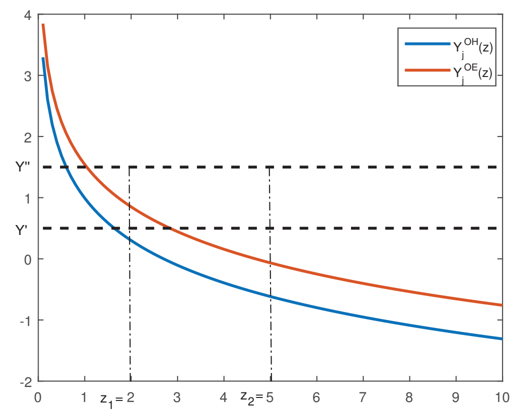
Figure 3: Model's predictions.

Affiliate size, export status, and the timing of entry.

(a) Exporters vs non-exporters

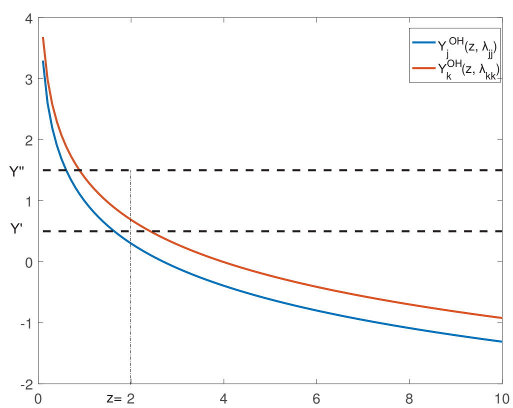


(b) Early vs late exporters

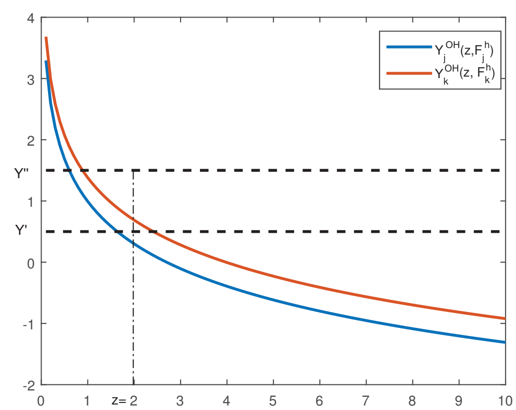


Host market characteristics and the timing of entry.

(c) Affiliate size



(d) Entry costs



in country k.

4 Calibration

We calibrate the model to match the expansion of US MNEs during the period 1987-2011, in the top-ten host countries for US FDI (Brazil, Canada, China, France, United Kingdom, Germany, Ireland, Japan, Mexico, and Singapore). We set the values of preference and technology parameters using estimates from the literature and direct observations from the data. Then, we jointly calibrate the rich set of barriers to MNE expansion included in the model to match static and dynamic moments from the BEA data.

4.1 Procedure

We set the elasticity of substitution $\eta = 5$, in line with estimates in the literature (Broda and Weinstein, 2006). We need to set the time preference rate to $\rho = 0.1$ so that it does not violate the technical condition that ensures that the present discounted value of profits does not diverge ($\rho > \tilde{\mu}_j, \ \forall j$).[22] We assume that the distribution of firm productivities is Pareto, with location parameter normalized to $b = 1$ and shape parameter $\vartheta = 4.5$, consistent with estimates in the literature (Simonovska and Waugh, 2014).

We use data on expenditure-based real GDP growth across countries, from the Penn World Tables 9.0, to calibrate the composite shock process, for each country in our sample. The composite shock Y_j captures the effect on profits of both US aggregate productivity and aggregate demand in country j. We set the drift of the process, $\tilde{\mu}_j$, to match real GDP growth in country j. Matching $\tilde{\sigma}_j$ to the standard deviation of real GDP growth, however, would generate too little volatility to induce reasonable firm dynamics. For this reason, we first set the standard deviation of US aggregate productivity, σ, to match the standard deviation of labor productivity among US firms, and the standard deviation of the aggregate demand shock in country j, σ_j, and its correlation with the US aggregate shock, γ_j, to match, respectively, country j's standard deviation of real GDP growth and its correlation with US GDP growth. We then use (8), together with $\eta = 5$, to recover the values of $\tilde{\sigma}_j$. To initialize the shock processes, we normalize the initial value of the US productivity shock to $Z(0) = 1$, and the US demand shock to $Q_{US}(0) = 1$. We then set $Q_j(0)$ to be equal to country j's GDP relative to US GDP, $\forall j$.

It remains to calibrate the preference shifters, λ_{ij}, and the parameters related to the costs of MNE expansion. These costs are: the fixed and sunk costs of affiliate opening, f_j^h and F_j^h, for $j = 1, ...10$; the fixed and sunk costs of affiliate exports, f_{jk}^e and F_{jk}^e, for $j = 1, ...10$, $k = 1, ...10, US$, and $k \neq j$; and the iceberg trade costs of affiliate exports, τ_{jk}, for $j = 1, ...10$, $k = 1, ...10, US$, and $k \neq j$.

Due to data limitations, we make some symmetry assumptions.[23] First, we assume that $\lambda_{ii} = 1$

[22] The value of $\rho = 0.1$ might appear high, but its interpretation includes economic magnitudes other than just the time preference rate. For example, if the model included an exogenous death rate, this variable would be added to the time preference rate and the technical condition would allow for a lower time preference rate. Since the solution of the model would be unchanged, we prefer not to add unnecessary parameters and rather to assume a high value for the time preference rate.

[23] As mentioned in Section 2.1, the BEA data do not record affiliate exports by destination country, except for the United States and for a handful of countries (Canada, Japan, and the United Kingdom) in benchmark-survey years.

and $\lambda_{ij} = \lambda_j \neq 1$, for $i = US$ and for all $j \neq i$. These taste shifters allow us to generate different market shares for domestic firms and US MNEs in a host country. Second, we assume that the fixed and sunk costs of affiliate exports are symmetric across all destination countries, except for the United States: $f^e_{jk} = f^e_j$ and $F^e_{jk} = F^e_j$, for $j, k = 1, ...10$, $k \neq j$, and $k \neq US$. Third, we assume that iceberg trade costs for destinations for which we do not have any bilateral affiliate export data are proportional to bilateral distance and to an exporter-specific dummy which is chosen to exactly match the aggregate export share from country j to all destinations.[24] Additionally, since aggregate demand grows over time at the rate μ_j, we assume that the fixed and sunk costs of MNE activities in each host country j also grow at the deterministic rate μ_j. Hence, we need to calibrate the initial values of the fixed and sunk costs for each host country. Without this growth adjustment, in the long run, frictions to MNE activities would become irrelevant to the firm's decision and all firms would become MNEs with affiliates in every country. Finally, we take wages as exogenous and we set them to match real GDP per unit of equipped labor, from Klenow and Rodríguez-Clare (2005), an average over the period 1995-2000. Appendix Table D.1 shows the results for each of the top ten host countries for US FDI.

We are left with 117 parameters to calibrate, for which we target 117 moments from the data. Even though the model does not have a one-to-one mapping from each parameter to each moment, and parameters are jointly calibrated, because of the model's closed-form solutions, it is relatively easy to isolate the moment that drives the identification of a given parameter. More precisely, the intensive margin of exports, given by export sale shares, drives the identification of the iceberg trade cost τ_{jk}, while affiliate entry rates and the share of MNE affiliates in each country help identify the sunk and fixed MNE entry costs, F^h_j and f^h_j, respectively.[25] Similarly, export entry rates and the share of exporting affiliates help identify the sunk and fixed export costs, F^e_j and f^e_j, respectively. Finally, the ratio of affiliate horizontal sales in country j to parent US sales helps identify the taste shifter λ_j.

We choose the values of the parameters that best fit the data moments, for each country. To this end, we simulate the model 100 times, each time for a different realization of the vector of aggregate shocks. Each simulation amounts to solving the model for 1,000 firms and 30 years. Computationally, this entails solving $N + N^2$ systems of four equations in four unknowns, for each firm and time period, as well as solving for the equilibrium price index every period.

[24] The distance elasticity is calculated by running a standard gravity equation with two sets of fixed effects and assuming that the trade elasticity is 4, consistent with the calibrated value of η.

[25] Since the share of affiliates, affiliate entry rates, and affiliate exit rates are linearly dependent, it is enough to target two out of the three moments. In the calibration, we target the share of affiliates and affiliate entry rates, and leave affiliate exit rates as non-targeted moments.

4.2 Model's fit

Table 3 reports simulated and data moments taking averages across the top-ten host countries for US FDI and across years. Appendix Tables D.4-D.9 report the full set of simulated and data moments, while Appendix Tables D.2 and D.3 show the calibrated parameters by country. We construct moments from the data using the sample of all affiliates operating in the top-ten host countries for US FDI. This sample includes 83,214 affiliate-year observations, which account for 68.8 percent of all sales by foreign affiliates of US MNEs.

Table 3 shows that the model matches quite well both the static and dynamic targeted moments. We also include in the table three sets of non-targeted moments: moments related to affiliate size advantage, MNE sorting patterns, and exit moments.

The moments capturing the affiliate size advantage are related to the analytical predictions of the model described in Section 3.4. First, Proposition 1 implies that, controlling for the affiliates' host market, affiliates that export have larger horizontal sales than affiliates that do not export. In the data, the average horizontal sales of an affiliate that exports from birth are 6.3 times larger than the average horizontal sales of an affiliate that never exports, averaging across affiliates' host markets. Our calibrated model generates an exporter premium among MNE affiliates of around seven. Similarly, the model predicts that affiliates that start exporting earlier in their life have larger horizontal sales than affiliates that start exporting later. In the data, the average horizontal sales of an affiliate that starts exporting in its first year of life are 3.7 times larger than the average horizontal sales of an affiliate that starts exporting after its first year of life, averaging across affiliates' host markets (see also Appendix Figure D.1). Our calibrated model generates an early-exporter premium of 5.5. Appendix Table D.9 shows results by country.

Second, the model has predictions about the expansion patterns of a MNE. Proposition 2 implies that MNEs open their largest affiliates first. In the data, on average, the horizontal sales of a first affiliate of a MNE are 2.6 times larger than the horizontal sales of the MNE's subsequent affiliates. The model generates a first-affiliate size premium of 2.1.[26] Proposition 1 also has implications about the sorting patterns of MNE entry. Similarly to Eaton et al. (2011) for exporters, the model predicts that the largest MNEs (in terms of US sales) should enter more markets and less popular markets. Indeed, as moments 5.1 and 5.2 show, the data corroborate the model's predictions: the larger the average sales of the MNE in the United States, the more markets the MNE enters; the

[26] Proposition 2 also states that MNEs first open affiliates located in countries where entry is less costly. Even though measuring entry costs directly in the data is difficult, we proxy them with the commonly used World Bank Doing Business indicators and provide suggestive evidence that supports the model's prediction, in Appendix Table D.10.

Table 3: Moments: model versus data, averages.

	data	model
Targeted Moments		
1. Static moments: intensive margin		
1.1 Affiliate sales share to host country	0.026	0.026
1.2 Affiliate sales share to the US	0.139	0.139
1.3 Affiliate sales share to third countries	0.288	0.296
1.4 Affiliate sales share to Canada	0.015	0.014
1.5 Affiliate sales share to the UK	0.069	0.087
1.6 Affiliate sales share to Japan	0.033	0.026
2. Static moments: extensive margin		
2.1 Share of MNEs with affiliates in j	0.287	0.283
2.2 Share of affiliates in j exporting to US	0.566	0.566
2.3 Share of affiliates in j exporting to third countries	0.650	0.646
3. Dynamic moments: entry		
3.1 Share of MNEs opening affiliates in j	0.035	0.021
3.2 Share of affiliates in j that start exporting to the US	0.030	0.024
3.3 Share of affiliates in j that start exporting to third countries	0.031	0.027
Non-Targeted Moments		
4. Static moments: size advantage		
4.1 Exporter size advantage	6.27	6.97
4.2 Early-exporter size advantage	3.68	5.54
4.3 First-affiliate size advantage	2.57	2.13
5. Static moments: MNE sorting		
5.1 Elasticity of average sales in US w.r.t. # of markets entered	0.736	0.816
5.2 Elasticity of average sales in US w.r.t # of firms entering multiple markets	-0.424	-0.722
6. Dynamic moments: exit		
6.1 Share of MNEs shutting down affiliates in j	0.113	0.083
6.2 Share of affiliates in j that stop exporting to the US	0.025	0.042
6.3 Share of affiliates in j that stop exporting to third countries	0.027	0.037

Note: Averages across host countries and years. Data moments for Japan, Canada, and the United Kingdom are averages over benchmark-year surveys only. Shares' denominators are: in 1.1, US parent's sales; in 1.2-1.6, total horizontal sales of affiliates in j; in 2.1, the total number of MNEs; in 2.2 and 2.3, the total number of affiliates in j; in 3.1, total number of MNEs in period before entry; in 3.2 (3.3), total number of affiliates in j in period before export entry into US (third countries); in 6.1, total number of affiliates in j in period before exit; and in 6.2 (6.3), the total number affiliate in j that export to the US (third countries) in the period before stopping the activity. In 4.1, exporter size advantage refers to the average size of exporting MNE affiliates relative to the average size of non-exporting MNE affiliates, an average across countries and years. In 4.2, affiliate early-exporter size advantage refers to the average size of exporting MNE affiliates that start exports in their first year of life relative to the average size of exporting MNE affiliates that start exports after their first year. In 4.3, first-affiliate size advantage refers to the ratio of the size of first foreign affiliate of a MNE (relative to GDP in the affiliate host market) to the size of subsequent foreign affiliates of the same MNE (relative to GDP in the affiliate host market), an average across MNEs and years. For moments in 4., size refers to horizontal affiliate sales. Calculations in the calibrated model trim the upper and lower 10th decile of the simulated firm-level data.

Table 4: Calibrated MNE costs: shares of sales and monetary values, average across host countries.

	As % of sales			In thousands $
	5th	50th	95th	
Sunk affiliate entry cost F_j^h, as % of US parent sales	0.05	0.02	0.00	161.0
Fixed affiliate entry cost f_j^h, as % of horizontal sales	18.7	12.2	2.30	1,525
Sunk export cost F_{jk}^e, as % of horizontal sales				
To United States	0.33	0.16	0.03	15.60
To other destinations	3.00	0.99	0.18	97.30
Fixed export cost f_{jk}^e, as % of exports sales				
To United States	12.7	10.1	1.70	98.50
To other destinations	12.8	11.6	3.30	814.6

Note: For F_j^h and f_j^h, we consider variables in the year of affiliate entry. For F_{jk}^e and f_{jk}^e, we consider variables in the year the affiliate first exports to the destination. Percentiles are with respect to affiliate sales in the calibrated model. Cost shares in the model are converted into thousands of US dollars using sales values for the median affiliate in each of the top-ten host countries included in the calibration, from the BEA, and averaged across host countries. For confidentiality purposes, median sales are an average of the 9 observations around the median.

smallest MNEs in terms of US sales enter only the most popular market (i.e. the one that all other MNEs also enter), while the largest MNEs in terms of US sales enter also the least popular markets. Table 3 shows that the calibrated model captures the magnitude of the elasticity of moment 5.1 very accurately, but overestimates the one of 5.2. Appendix Figure D.2 illustrates these sorting patterns by number of host markets.

Finally, as shown in the last panel of Table 3, the exit rates in the model are close to the ones we observe in the data. The model slightly under-predicts affiliate exit, and over-predicts exit from export markets.

4.3 The costs of MNE expansion

We now evaluate the magnitude of the costs of MNE expansion in time and space. Since model-based magnitudes are hard to interpret, in Table 4 we express the calibrated MNE costs as shares of firm revenues and in monetary values. Appendix Tables D.11 and D.12 report costs as shares of revenues by host country, while Appendix Table D.13 shows sales, in US dollars, as observed in the BEA data, by host country.

On average, opening an affiliate involves spending a very low share of a firm's US parent revenues. An affiliate's fixed operating costs range from about two percent of the affiliate's horizontal sales, for the largest affiliates, to about 19 percent, for the smallest affiliates. In monetary terms, affiliate export operations appear less costly than affiliate horizontal operations. As expected, affiliate exports to the United States have lower costs than exports to other destination markets, especially in terms of sunk costs. This result is intuitive as the United States is the origin country of the affiliates.

The relative magnitudes of different fixed and sunk costs of affiliate operations and affiliate exports are informative about the heterogeneity of the frictions to MNE activities, both by country and by type of affiliate sales. Table 5 shows that opening affiliates in Brazil, Canada, China, Japan and Mexico is much more costly than starting to export from them. Favorable trade regimes, such as NAFTA, or tax policies favoring exports but restricting horizontal FDI, as in China, can rationalize this pattern. Ratios lower than one for countries like Ireland and Singapore may be explained by favorable "tax-haven"-like policies that attract FDI.

There is a large amount of heterogeneity in the fixed costs of setting up an affiliate relative to the fixed cost of exporting from it. Nonetheless, all the calibrated ratios are above one, indicating that the operating cost of horizontal activities is higher than the operating cost of exporting in all host countries.

The third and fourth columns in Table 5 shows the peculiarity of the United States as an export destination for affiliates of US MNEs. If the United States were an "average" export market for affiliates, the reported ratios would be close to one. This is true only for the sunk export cost paid by affiliates located in the United Kingdom where the United States appears similar to other export destinations. Surprisingly, the United States seems to be a more expensive market for affiliates located in Canada and Mexico. This may be because, for example, affiliates located in Canada also export large magnitudes to Mexico, which is also part of NAFTA, and exports to Mexico may be associated with lower costs than exports to the United States.

5 Quantitative Analysis

Armed with the calibrated model, we explore the implications of counterfactual scenarios on aggregate firm dynamics. Since the top-ten host countries for US FDI in our sample include the United Kingdom, Ireland, Germany, and France, we use as our counterfactual exercise the possibility of the United Kingdom abandoning the European Union (EU), "Brexit". Potential implementations of Brexit have as a common element the increase in export costs between the United Kingdom and other countries in the EU. In the model, this increase can be captured either as an increase in the

Table 5: Calibrated MNE costs: by type, destination, and host country.

	F_j^h/F_j^e	f_j^h/f_j^e	$F_{jk}^e/F_{j,US}^e$	$f_{jk}^e/f_{j,US}^e$
	(1)	(2)	(3)	(4)
Brazil	117	4.80	4.20	0.82
Canada	NaN	4.95	0.00	0.63
China	2.10	1.80	0.45	0.75
France	0.31	3.10	470	2.26
United Kingdom	0.01	2.80	1.01	1.91
Germany	0.15	2.80	30.2	2.45
Ireland	0.43	1.90	NaN	0.95
Japan	42.2	6.40	36.4	1.21
Mexico	12.6	4.70	0.06	0.70
Singapore	0.03	1.07	6.21	0.84

Note: In column 1 (column 2), the denominator is an average of the sunk (fixed) costs of exporting to the United States and to any other destination k, from country j. In column 3 (column 4), the ratio is between the sunk (fixed) export cost from j to any other destination k, relative to the costs of exporting to the United States. "NaN" are ratios in which the denominator is almost zero.

iceberg trade cost, $\tau_{UK,j}$, an increase in the fixed export cost, $f_{UK,j}^e$, or an increase in the sunk export cost, $F_{UK,j}^e$, for $j =$ France, Germany, Ireland. We can think about "shallow Brexit" scenarios, where only one type of friction is affected, or "deep Brexit" scenarios, where all the export barriers increase simultaneously. Additionally, in the Brexit context, we evaluate the importance of including the endogenous price response to the various shocks by comparing the aggregate dynamics of the model with and without endogenous prices. In our model, the magnitude of the price change is also informative about changes in the real wage—i.e., our welfare measure.

In a second counterfactual, we raise the cost of MNE activities in the United Kingdom, captured in our model by the per-period cost of MNE operations, f_{UK}^h, or alternately, by the sunk entry cost, F_{UK}^h. We use this exercise to evaluate quantitatively the role played by the compound option structure of our model. To such end, we evaluate the response to a cost shock in a calibrated model with and without export platforms.

Overall, our quantitative exercises point to the importance of including dynamics, together with a rich spatial structure, when evaluating the response of MNE expansion to shocks.

For each counterfactual exercise, we simulate the model for 30 periods and impose a permanent change in one or more of parameters at $t = 15$.

5.1 Brexit

First we increase, one at the time, the barriers to export from (to) the United Kingdom to (from) the EU countries in our sample ("shallow Brexit"): $\tau_{UK,j}$, $f^e_{UK,j}$, and $F^e_{UK,j}$, for j = Ireland, Germany, and France. In order to make the results of the three exercises comparable, we increase each trade friction by an amount that results in a 20 percent increase in the total per-period cost of FDI, $\left(f^e_{UK,j} + \rho F^e_{UK,j}\right) \tau^{\eta-1}_{UK,j}$. For "deep Brexit", we increase all export barriers at once. [27]

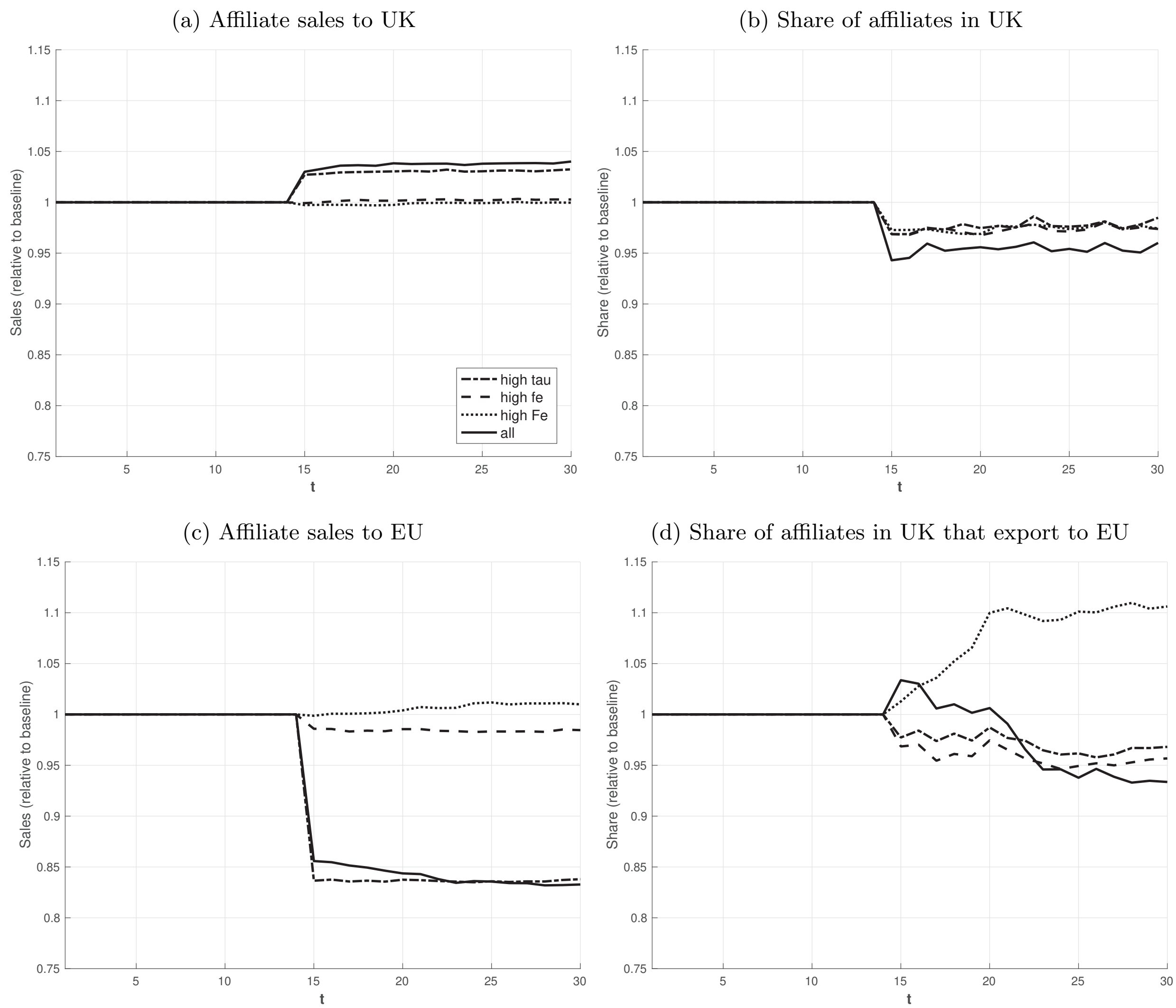
Intuitively, increasing trade barriers between the United Kingdom and other EU countries has three main effects. First, exporting from the United Kingdom to the EU becomes more costly, so that export sales from UK-based affiliates to countries in the EU decline, decreasing the incentive to open affiliates in the United Kingdom due to the smaller, and more costly, available network of export destinations. Analogously, exporting from the EU to the United Kingdom also becomes more costly, so that export sales from affiliates in EU countries to the United Kingdom also decline, decreasing the incentive to open affiliates in those countries. These are static, partial equilibrium effects. In addition, increases in trade costs affect the affiliate export band of inaction, which in turn affects affiliate export entry and exit —an effect only present in dynamic models. Finally, increases in trade frictions have the effect of raising the price index in the destination countries, encouraging more export entry. Our quantitative results combine the effects of these three forces.

Figure 4 shows the effects of "shallow Brexit" on affiliates of US MNEs in the United Kingdom. Results are presented as deviations from the baseline scenario—the calibrated model. Before the shock, about half of US MNEs had affiliates located in the United Kingdom, as shown in Appendix Table D.6. All three "shallow Brexit" shocks cause a permanent decrease in the share of US MNE affiliates in the United Kingdom of around three percent (Figure 4b). The cumulative increase of all frictions ("deep Brexit") drives a decrease in the share of US affiliates located in the United Kingdom of five percent. Naturally, the affiliates that exit are the smallest and account for only a few percentages of US affiliate sales in the United Kingdom. This observation, together with the contemporaneous increase in the price index in the United Kingdom, shown in Figure 7, explains the somewhat counterintuitive result of Figure 4a, where affiliates sales to the United Kingdom increase when variable trade costs increase. The effects of changing trade costs on horizontal activities in the United Kingdom are small, since trade frictions affect those activities only indirectly through the compound option.

The lower panels of Figure 4 show the effect of "shallow Brexit" on the export sales and export participation rates of UK-based affiliates of US MNEs to Ireland, Germany, and France. These plots

[27] At $t = 15$, the model's matches the shares of affiliates of US MNEs in the United Kingdom observed in the data.

Figure 4: Brexit: US MNEs' affiliates in the United Kingdom.



Note: "high X" refers to an increase in the barrier X from/to the United Kingdom to/from country j ("shallow Brexit"). "All" refers to increasing all three export barriers from/to the United Kingdom to/from country j at once ("deep Brexit"). Country j refers to Ireland, Germany, and France.

illustrate how different frictions to MNE activities have very different quantitative effects on affiliate exports and participation rates, even if the changes in those frictions are associated with the same increase in the per-period cost of FDI. More precisely, the observed decline in export sales, when either $f^e_{UK,j}$ or $\tau^e_{UK,j}$ increase, comes from affiliates that stop exporting. The decline in export sales also includes an intensive-margin decline for the case of a shock to $\tau_{UK,j}$. Consequently, the change that has the highest impact on affiliate export sales is the increase in the per-period iceberg trade cost. An increase in $\tau_{UK,j}$ corresponding to a 20 percent increase in the cost of FDI produces a 15 percent decline in UK-based affiliates' export sales, a much larger decline than the one produced by increases in fixed export costs. Conversely, an increase in the sunk export cost, $F^e_{UK,j}$, produces a small increase in affiliate export sales. As Appendix Figure E.5 shows, an increase in the sunk export cost increases the export band of inaction, driving a decline in both affiliate export entry and exit rates. The decline in the exit rate is the most pronounced, giving rise to the increase in affiliate export sales observed in Figure 4c, and in the share of exporting affiliates observed in Figure 4d. Except for the case of an increase in the sunk export cost, the Brexit shock has the effect of reducing the share of affiliates that export. The increase in the fixed export cost produces the largest decline in the export participation rate because this cost is intimately related to the affiliates' decision to exit a market. The difference in the magnitude and direction of the effects when either sunk, fixed, or variable costs change highlights the importance of including dynamics in models of MNE behavior. Static models cannot distinguish between sunk and fixed costs, yet MNEs respond very differently to changes in these types of costs.

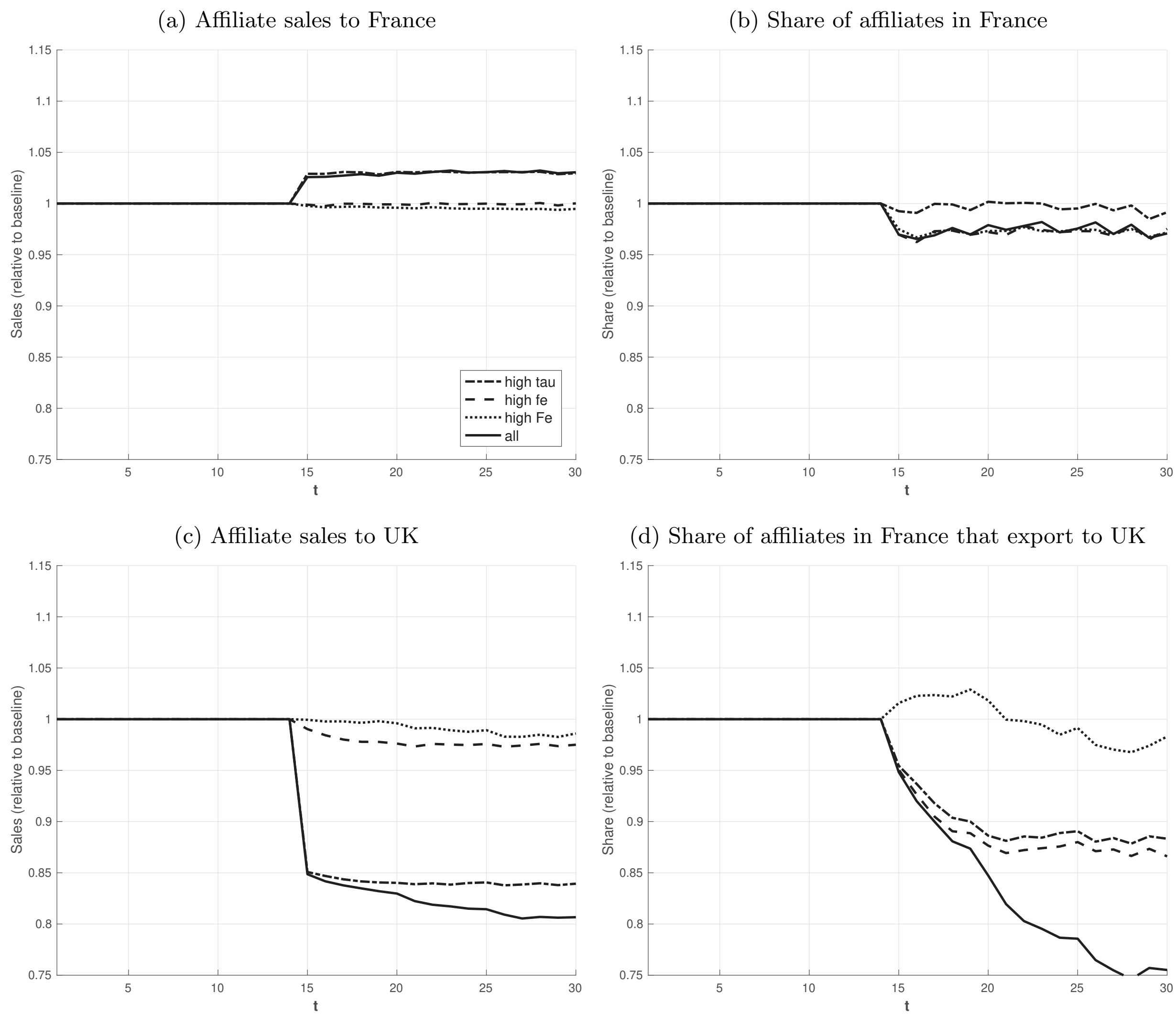
The case of "deep Brexit" highlights the differences between the aggregate dynamics after a shock in the short and in the long run. In particular, in the short run the effect of the increase in sunk export costs dominates and the share of affiliates in the United Kingdom that export to EU increases. Fifteen periods after the shock, this share decreases by around seven percentage points relative to the pre-shock levels.

The results for US affiliates located in France, Ireland, and Germany are qualitatively similar to the results for US affiliates located in the United Kingdom. There are, however, some important quantitative differences. Figure 5 shows the results for affiliates of US MNEs located in France. We relegate to Appendix Figures E.3 and E.4 the results for Ireland and Germany.

As expected, higher trade costs between France and the United Kingdom reduce the incentives to locate in France, and the share of affiliates of US MNEs located in France declines. As already seen for the United Kingdom, this decline in the extensive margin is accompanied by an increase in the intensive margin of horizontal sales in France for the scenario in which $\tau_{UK,FR}$ increases. This increase is driven by the corresponding increase in the price index in France. Naturally, the share of US affiliates in France that export to the United Kingdom, as well as their export sales, drop after the Brexit shock in almost all specifications. An increase in the sunk export cost generates a non-monotonic response of the share of exporting affiliates, which first increases (due to a decline in exit rates) and then decreases.

Summing up, the results in Figures 4 and 5 show that increasing a variable, fixed, or sunk cost of exporting has different *quantitative* effects on aggregate firm dynamics. Even though the increase

Figure 5: Brexit: US affiliates in France.



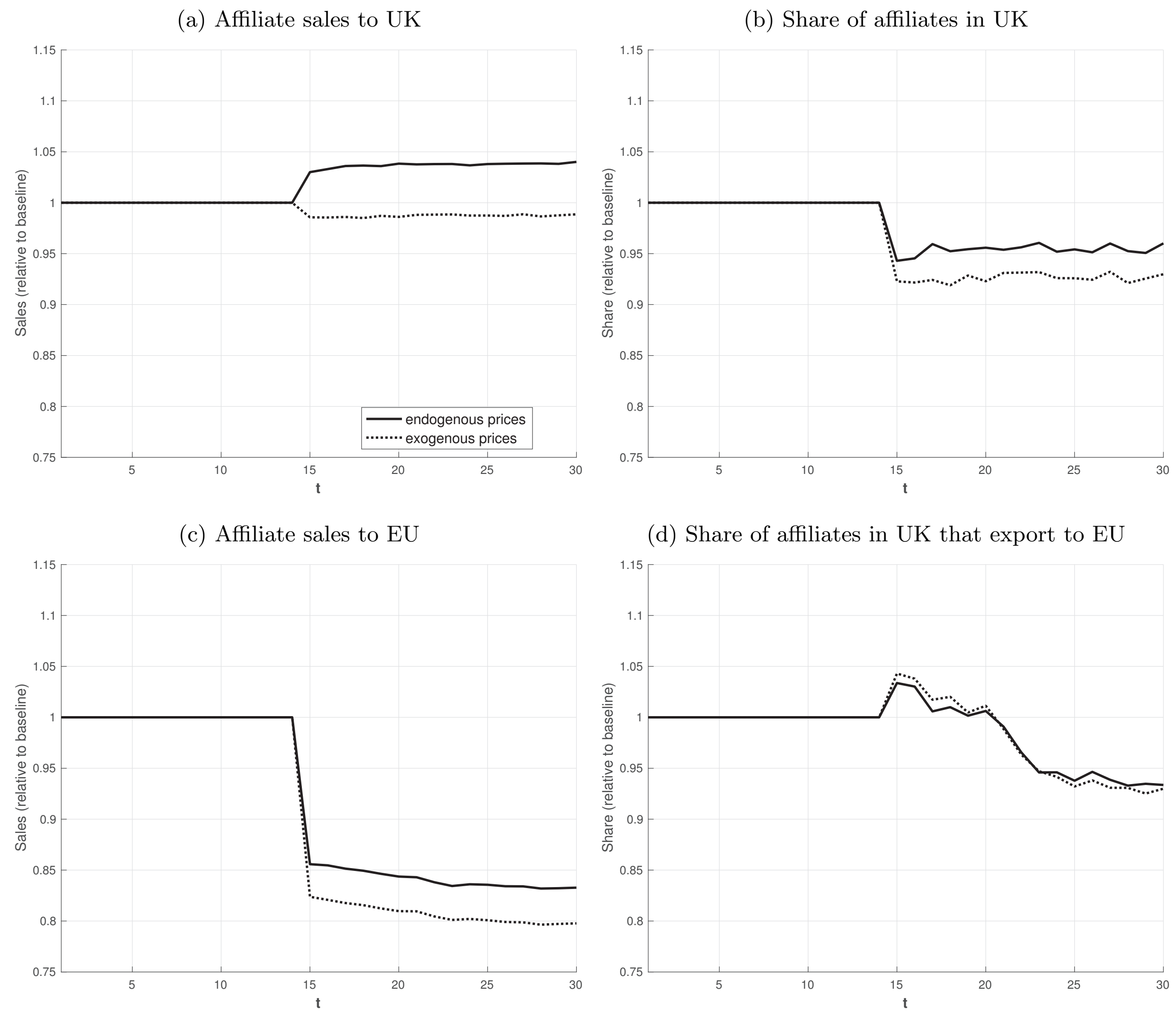
Note: "high X" refers to an increase in the barrier X from/to the United Kingdom to/from country j ("shallow Brexit"). "All" refers to increasing all three export barriers from/to the United Kingdom to/from country j at once ("deep Brexit"). Country j refers to Ireland, Germany, and France.

in the per-period cost of FDI is the same in all cases, the type of trade barrier that changes matters for both the extensive and intensive margins of MNEs' decisions, and in turn, for aggregate dynamic responses.

Thanks to the tractability of the model, we can solve numerically for the aggregate price index in each period, for each country. In this way, the results of our counterfactual exercises incorporate the effects of changes in the price indexes on firms' decisions. How important are these price effects *quantitatively*? Are they strong enough to affect the aggregate dynamics of MNEs? In Figure 6, we show the dynamics of aggregate outcomes after the "deep Brexit" shock, under endogenous and exogenous prices. It is clear that the endogenous response of prices acts as a buffer to the decrease in affiliate sales, both horizontal and exports, as well as to the decrease in the share of US MNEs operating in the United Kingdom. In the case of horizontal sales, shown in Figure 6a, the effect is strong enough to reverse the pattern from a two-percent decrease to a three-percent increase.

Appendix Figure E.1 shows results with exogenous prices for US MNE affiliates in the United Kingdom, for each of the Brexit counterfactual exercises.

Figure 6: Brexit: US affiliates in the United Kingdom. Endogenous vs exogenous prices.

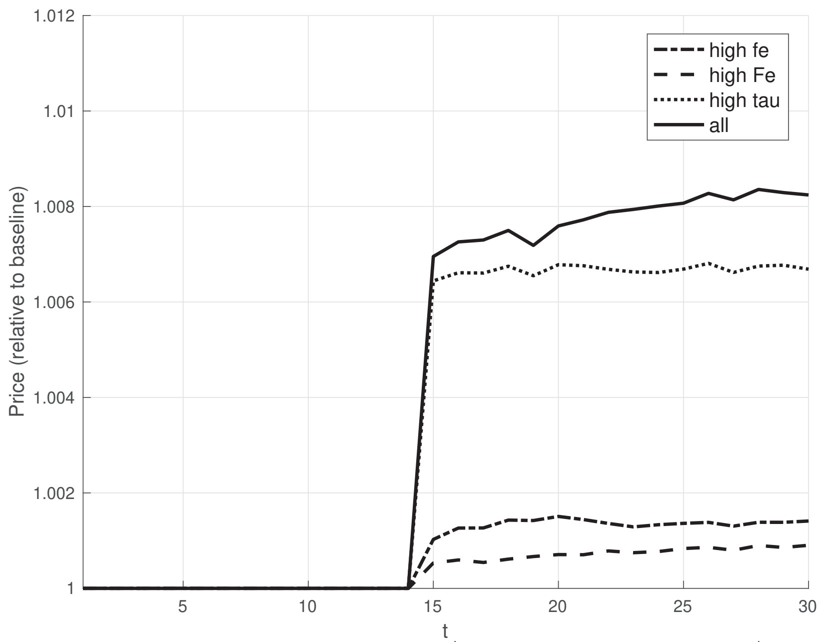


Note: The shock refers to increasing all three export barriers from/to the United Kingdom to/from country j at once ("deep Brexit"). Country j refers to Ireland, Germany, and France.

It is straightforward to evaluate the welfare losses from the different Brexit scenarios using the counterfactual price changes. Figure 7 shows that, as expected, the largest welfare loss is experienced under "deep Brexit," with a decrease in real income that goes from 0.7 percent in the short run to more than 0.8 percent after fifteen years. In the context of "shallow Brexit," an increase in variable trade costs produces the highest loss, while an increase in sunk export costs produces the smallest loss. These magnitudes are not small given that our model is restricted to the behavior of MNEs from the United States. The increase in trade barriers would presumably affect also local exporters and other non-US MNE exporters. Appendix Figure E.2 shows the change in prices after each Brexit shock, for each of the countries involved. As expected, changes in prices–and corresponding welfare losses–are highest for Ireland, whose economy is deeply connected to the UK economy.

When interpreting our results, two remarks are in order. On the one hand, the compound option structure, together with Assumption 1, implies that increases in bilateral frictions only affect the incentives to operate affiliates in continental Europe through changes in the cost of accessing the export network available from a given host country. The model does not produce reallocations across host countries, and this feature may have the effect of overstating the losses from increasing MNE frictions. On the other hand, since we assume that wages are exogenous, the aggregate price effects are stronger than in the case of endogenous wages, and this may understate the losses from increasing MNE frictions.

Figure 7: Brexit: Price changes in the United Kingdom.



Note: "high X" refers to an increase in the barrier X from/to the United Kingdom to/from country j ("shallow Brexit"). "All" refers to increasing all three export barriers from/to the United Kingdom to/from country j at once ("deep Brexit"). Country j refers to Ireland, Germany, and France.

Our Brexit counterfactual illustrates the importance of considering the global structure of the MNE in time *and* space for accurately assessing the consequences of shocks to the costs of MNE expansion. In a static model, one would obviously not be able to distinguish between short and long run outcomes, but more importantly between the effects of one-time versus per-period costs. Additionally, in a dynamic model in which MNEs could only serve the host market of operation, one would erroneously conclude that Brexit, in the form of higher trade barriers, would not have any impact on the behavior in of MNEs. We explore in more detail the role played by export platforms in the next counterfactual, where we evaluate the role of the compound option structure of the model.

5.2 The role of the compound option

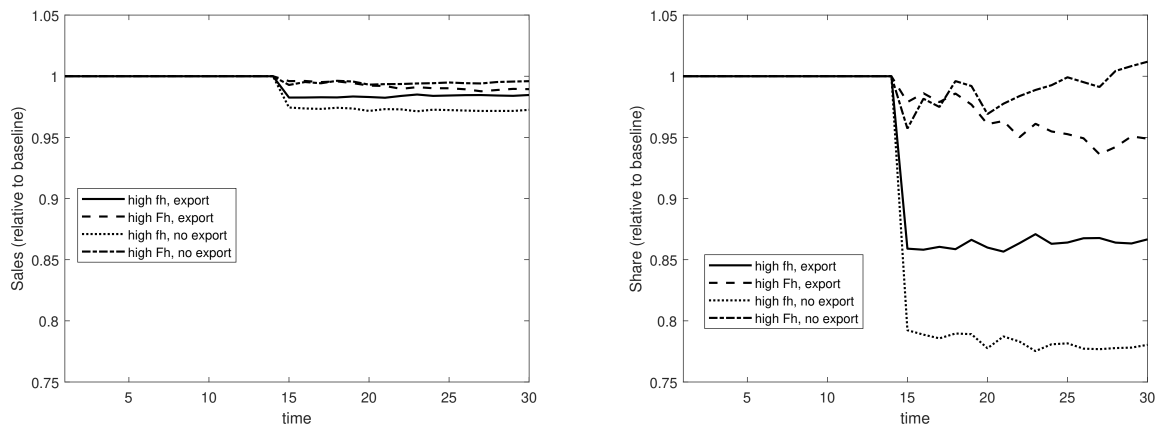
We now evaluate quantitatively the role played by the compound option structure of our model. To such end, we analyze the effects of an increase in the barriers to MNE activities in a model with and without the compound option—or analogously, a model with and without MNE affiliate exports. We increase, alternately, the per-period fixed cost and the one-time sunk cost of MNE activities in the United Kingdom, f^h_{UK} and F^h_{UK}, in an amount such that, in either case, $f^h_{UK} + \rho F^h_{UK}$ increases by 20 percent.

In our counterfactual model, MNE affiliates only sell in their host country. We calibrate this alternative model by targeting the moments which are common to the full model with exports (moments 1.1, 2.1, and 3.1 in Table 3). [28]

Figure 8: The role of the compound option: US MNE affiliates in the United Kingdom.

(a) Affiliate sales to UK

(b) Share of affiliates in UK



Note: At $t = 15$, the per-period fixed cost (one-time sunk cost) of MNE activities in the UK, f^h_{UK} (F^h_{UK}), increases by an amount such that $f^h_{UK} + \rho F^h_{UK}$ increases by 20 percent. "Exports" refers to the full model where MNE affiliates can export, while "no exports" refers to the calibrated model where MNE affiliates cannot export. Results are shown as deviations from the calibrated models with and without export platforms.

Figure 8 shows the dynamics of horizontal affiliate sales and the share of US affiliates in the United Kingdom after the increase in f^h_{UK} and F^h_{UK}, alternately. Results are shown as deviations from the respective calibrated models with and without the compound option. In the model with only horizontal sales (dashed lines), MNE affiliates do not have the option of exporting part of their output. Hence, the incentives to open an affiliate in a country coming from the possibility of using that host country as an export platform are precluded. Without the possibility of exporting from the United Kingdom to other markets, an increase in per-period costs to MNE activities in the United Kingdom would decrease the presence of US affiliates in the country by 25 percent more (in the short run) than in the case in which exports are possible. In the case of an increase in sunk MNE costs, while differences between both models are not large in the short run, the models have different predictions for the long run. While not having the possibility of exporting from the United Kingdom steadily decreases the share of US MNEs there, the inclusion of the possibility of exporting reverses the initial decrease in the share of US MNEs in the United Kingdom—after 15

[28] The calibrated model without export platforms does not fit the relevant data moments as well as the baseline model. While the share of US MNEs with affiliates in the United Kingdom is 53.7 (55.4) percent in the model without exports (data), the share of MNEs opening affiliates the United Kingdom is 2.2 (6.9) percent in the model (data), and the share of UK affiliates shutting down is 8.8 (4.5) percent in the model (data).

periods from the shock, this share is even higher than in the baseline.

The results in Figure 8 demonstrate the importance of including a rich dynamic spatial structure when evaluating the response of MNE expansion to shocks. These results also highlight the importance of distinguishing between per-period and one-time costs, which cannot be done in static models.

6 Conclusions

This paper studies the expansion patterns of the multinational enterprise (MNE) in time and space. Using a long panel of US MNEs, we document stylized facts which guide us in the development of a dynamic model of the MNE that is tractable and, at the same time, rich enough to capture the spatial complexity of MNE activities observed in the data. The model features heterogeneity in firm productivity, persistent aggregate shocks, and a realistic structure of MNE costs. Importantly, MNE affiliates can decouple their locations of production and sales, and endogenously choose to enter or exit both the host and the export markets. We introduce a compound option formulation for the dynamic problem of the MNE, which is novel to the literature. Our counterfactual exercises reveal that the compound option structure is important for understanding the reallocation of MNE activity in time and space after a shock. These exercises also reveal that the nature of the frictions to MNE activities (variable, fixed, or sunk) is important for understanding aggregate firm dynamics after a shock.

The application of the compound option structure to the complex decisions of the MNE appears a natural starting point, since MNE location and export choices happen over time and they are likely to be affected by uncertainty in demand and other market characteristics. The compound option formulation, however, can also prove useful for problems in other contexts. For instance, problems related to global sourcing decisions, which are likely to occur over time and under uncertainty, are good candidates. It is sensible to imagine a set up in which making an investment to source an input from a given country opens up the possibility of sourcing other, more upstream, inputs from a different location. This is an avenue of research worth exploring in the future.

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Appendix

A Data Description

Reporting thresholds. The BEA collects firm-level data on the operations of US multinational enterprises (MNEs) in its annual surveys of US direct investment abroad. All US-located firms that have at least one foreign affiliate and that meet a minimum size threshold are required by law to respond to these surveys. These minimum size thresholds are in terms of affiliate sales and differ over time. In general, reporting thresholds increased in recent years, reaching US$60 million by 2011. Additionally, benchmark survey years (i.e., years in which the survey is more comprehensive), which occur every 5 years, have lower reporting thresholds. Table A.1 shows the reporting thresholds for the years in our sample.

Table A.1: BEA minimum survey exemptions levels.

survey year	Minimum exemption levels (in US$ millions)	survey year	Minimum exemption levels (in US$ millions)
1987-88	10	2000-03	30
1989	**3**	**2004**	**25**
1990 -93	15	2005-07	40
1994	**3**	2008	60
1995-98	20	**2009**	**25**
1999	**7**	2010-11	60

Note: Exemption levels are for majority-owned foreign affiliates. Benchmark survey years are **in bold**.

Tax havens. Our sample contains affiliates that do not operate in tax haven countries. Affiliates in tax haven countries are likely to open for different reasons than production purposes, and to be subject to different cost structures than affiliates in non-tax haven countries. We exclude countries defined as tax havens by Gravelle (2015), except for Ireland, Switzerland, Hong Kong, and Singapore that meet some of the criteria for tax haven status but also have a substantial amount of US MNE production. Table A.2 reports the list of countries that we exclude from our sample.

Industry classification. Each foreign affiliate is assigned an industry classification based on its primary activity according to the BEA International Surveys Industry (ISI) system, which closely follows the 3-digit Standard Industrial Classification (SIC) system. The BEA uses 3-digit SIC-

Table A.2: Tax haven countries excluded from our sample.

Anguilla	Turks and Caicos	Monaco
Antigua and Barbuda	US Virgin Islands	San Marino
Aruba	Belize	Maldives
Bahamas	Costa Rica	Mauritius
Barbados	Panama	Seychelles
British Virgin Islands	Bermuda	Bahrain
Cayman Islands	Macau	Cook Islands
Dominica	Andorra	Marshall Islands
Grenada	Channel Islands	Samoa
Montserrat	Cyprus	Nauru
Netherlands Antilles	Gibraltar	Niue
St. Kitts and Nevis	Isle of Man	Tonga
St Lucia	Liechtenstein	Vanuatu
St Vincent and Grenadines	Malta	Liberia

Note: From Gravelle (2015).

based ISI codes for years prior to 1999. From 1999 onward, they use 4-digit NAICS-based ISI codes. For consistency, we convert the NAICS-based codes to 3-digit SIC-based ISI codes for the relevant years.

Unit of observation. According to the BEA definition, an affiliate is a business enterprise operating in a given host country; it thus can operate several plants in different locations within the host country. The BEA rules permit consolidated reporting for distinct plants located in the same country that operate in the same narrowly defined industry or otherwise are integral parts of the same business operation. We consolidate observations of enterprises belonging to the same parent company and operating in the same country and 3-digit industry. We group these enterprises' activities together and refer to them as a single affiliate.

B Additional Facts

Table B.1: Affiliate-to-parent sales ratio by sales type, robustness. OLS.

	Dependent variable: affiliate-to-parent sales ratio					
	First affiliate	Subsequent affiliate	Non-GVC affiliate	GVC affiliate	Greenfield affiliate	M&A affiliate
	(1)	(2)	(3)	(4)	(5)	(6)
D(age=2)	-0.0014	0.0016	-0.004	0.0019		
	(0.0055)	(0.0012)	(0.0056)	(0.0019)		
D(age=3)	0.0001	0.0016*	-0.0142	0.0039***	0.0044	0.0050*
	(0.0066)	(0.0009)	(0.0141)	(0.0014)	(0.0029)	(0.0028)
D(age=4)	0.0003	0.0022**	-0.0157	0.0045***	0.0079**	0.0038
	(0.0067)	(0.0009)	(0.0147)	(0.0014)	(0.0031)	(0.0031)
D(age=5)	0.0033	0.0026***	0.0111	0.0032**	0.0069*	0.0034
	(0.0027)	(0.0009)	(0.0122)	(0.0013)	(0.0039)	(0.0031)
D(age=6)	0.0065	0.0030**	-0.0020	0.0057***	0.0066*	0.0053*
	(0.0040)	(0.0013)	(0.0053)	(0.0019)	(0.0037)	(0.0029)
D(age=7)	0.0018	0.0029**	-0.0009	0.0035**	0.0046	0.0047*
	(0.0032)	(0.0013)	(0.0040)	(0.0014)	(0.0028)	(0.0024)
D(age=8)	0.0013	0.0029**	-0.0064	0.0039***	0.0056*	0.0049**
	(0.0036)	(0.0013)	(0.0077)	(0.0012)	(0.0029)	(0.0022)
D(age=9)	0.0034	0.0042***	-0.0050	0.0051***	0.0089	0.0060***
	(0.0027)	(0.0015)	(0.0053)	(0.0014)	(0.0054)	(0.0021)
D(age=10)	0.0119	0.0039**	0.0279	0.0043***	0.0082	0.0056***
	(0.0094)	(0.0016)	(0.0291)	(0.0014)	(0.0058)	(0.0019)
log global employment	-0.0254	-0.0089	-0.0418	-0.0078	0.0133	0.0089
	(0.0208)	(0.0073)	(0.0347)	(0.0074)	(0.0158)	(0.0150)
Observations	17,360	20,720	10,320	27,760	2,214	3,564
R^2 (overall)	0.01	0.027	0.01	0.02	0.01	0.02

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. The dependent variable *affiliate to parent sales ratio* refers to affiliate sales relative to the domestic sales of the US parent. First affiliate refers to the first foreign affiliate opened by the parent, while subsequent affiliate refers to second or higher. GVC affiliate refers to affiliates with positive intra-firm trade, while non-GVC affiliate refers to affiliates with zero intra-firm trade. M&A affiliate refers to affiliates created through a merger or acquisition of an existing firm, while greenfield affiliate refers to a new firm. All specifications include affiliate and country-year fixed effects. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

Table B.2: Intensive and extensive margins of sales. OLS.

Dependent variable	Intensive margin of sale shares				Extensive margin of sale shares			
	horizontal sales		export sales		horizontal sales		export sales	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
affiliate age	-0.002	-0.012***	-0.005**	0.005***	0.00003	-0.001	0.014***	0.029***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.0006)	(0.003)	(0.002)
country-year fe	yes	yes	yes	yes	yes	yes	yes	yes
industry fe	yes	no	yes	no	yes	no	yes	no
affiliate fe	no	yes	no	yes	no	yes	no	yes
Observations	36,135	36,135	25,958	25,958	38,080	38,080	38,080	38,080
R-square	0.079	0.013	0.092	0.000	0.042	0.0001	0.081	0.036

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. In columns (1)-(4), the dependent variable is horizontal (export) sales, as a share of total affiliate's sales, for affiliates with positive horizontal (export) sales; in columns (5)-(8), the dependent variable is the share of affiliates with positive horizontal (export) sales. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

Table B.3: Intensive and extensive margins of sales, pure-type affiliates at birth. OLS.

Dependent variable	Intensive margin of sale shares				Extensive margin of sale shares			
	horizontal sales		export sales		horizontal sales		export sales	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
affiliate age	-0.014***	-0.096***	-0.036***	-0.021***	-0.046***	-0.044***	-0.059***	-0.038***
	(0.001)	(0.001)	(0.004)	(0.003)	(0.004)	(0.002)	(0.005)	(0.004)
country-year fe	yes	yes	yes	yes	yes	yes	yes	yes
industry fe	yes	no	yes	no	yes	no	yes	no
affiliate fe	no	yes	no	yes	no	yes	no	yes
Observations	19,910	19,910	3,590	3,590	19,910	19,910	3,590	3,590
R-square	0.147	0.020	0.288	0.133	0.245	0.099	0.268	0.125

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. In columns (1)-(4), the dependent variable is horizontal (export) sales, as a share of total affiliate's sales, for affiliates born with only horizontal (export) sales; in columns (5)-(8), the dependent variable is the share of affiliates born with only horizontal (export) sales. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

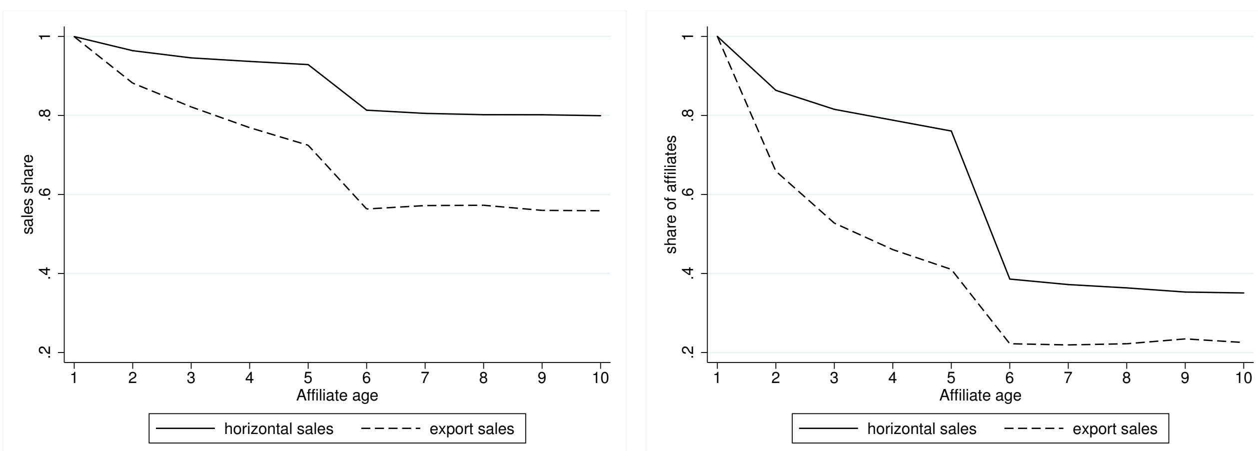
Table B.4: MNE shock structure, OLS.

Dependent variable	log of horizontal affiliate sales			
country-industry fixed effect	yes	yes	yes	yes
US GDP	yes	yes	yes	yes
Host country GDP	yes	yes	yes	yes
parent fixed effect	no	yes	yes	no
parent sales	no	no	yes	yes
affiliate fixed effect	no	no	no	yes
Adjusted R-squared	0.24	0.27	0.29	0.79

Notes: Sample of *all* affiliates born during the sample period. Number of observations: 153,773.

Figure B.1: Intensive and extensive margins of sales, by activity type. Pure-type affiliates at birth.

(a) Affiliate sales shares (intensive margin) (b) Share of affiliates (extensive margin)



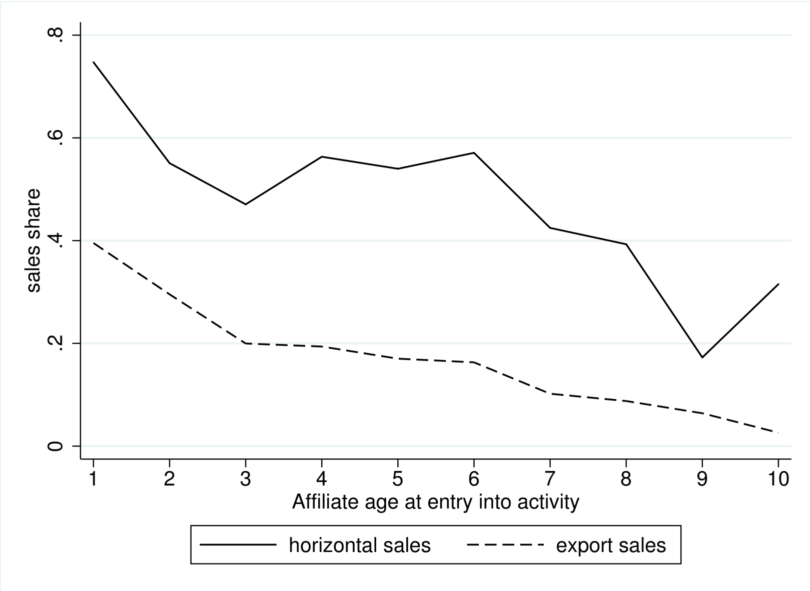
Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Horizontal and export sales refer to sales to the market where the affiliate is located, and to sales to markets outside the local market. (B.1a): average sales, as a share of total affiliate sales, include affiliates with positive horizontal and export sales, respectively, for the subset of affiliates with only horizontal and only export sales at birth, respectively). (B.1b): number of affiliates, as a share of the total number of affiliates, include affiliates with positive horizontal and export sales, respectively.

Table B.5: Unconditional and conditional probability of affiliate entry.

	Unconditional	Continent	Border	Language	Income	All
	(1)	(2)	(3)	(4)	(5)	(6)
Canada	0.021	***0.021***	–	0.023	***0.021***	–
		(0.525)	–	(0.000)	(0.553)	–
United Kingdom	0.025	0.027	***0.030***	***0.026***	0.026	***0.030***
		(0.000)	(0.143)	(0.292)	(0.008)	(0.143)
Germany	0.023	0.026	0.029	0.028	0.024	0.028
		(0.000)	(0.000)	(0.010)	(0.000)	(0.010)
Ireland	0.010	0.010	0.011	0.010	0.010	0.011
		(0.001)	(0.010)	(0.000)	(0.005)	(0.011)
China	0.027	0.037	0.050	0.048	0.051	0.057
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
France	0.021	0.024	0.028	***0.023***	0.022	0.029
		(0.000)	(0.000)	(0.018)	(0.000)	(0.000)
Brazil	0.016	0.022	0.027	***0.025***	0.023	***0.019***
		(0.000)	(0.000)	(0.063)	(0.000)	(0.614)
Singapore	0.016	0.023	0.044	0.017	***0.016***	0.045
		(0.000)	(0.000)	(0.000)	(0.300)	(0.000)
Mexico	0.024	0.029	***0.028***	0.034	0.031	***0.024***
		(0.000)	(0.620)	(0.000)	(0.000)	(0.961)
Japan	0.016	0.021	–	–	***0.016***	–
		(0.000)	(0.000)	(0.000)	(0.224)	(0.000)

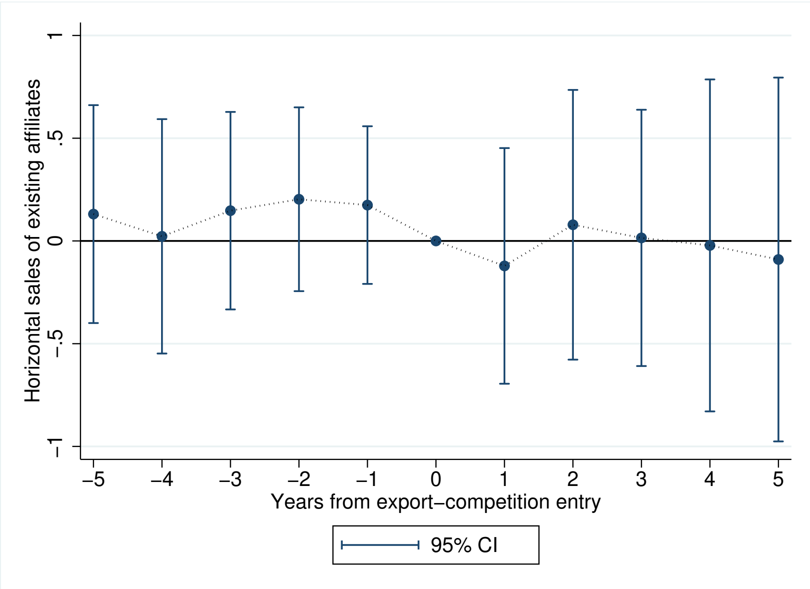
Note: Probabilities of affiliates' entry into the top-ten most popular destinations of US MNEs. Conditional probabilities refer to the probability of observing a MNE opening an affiliate in country i given that the parent already has an affiliate in a "similar" country. Column 6 refers to similarity in all the dimensions in columns 2-5. The sample is restricted to parents with at least two affiliates worldwide. P-values from tests of equality of the conditional and unconditional probabilities are in parentheses. Conditional probabilities in ***bold*** are not significantly different from the relevant unconditional probability.

Figure B.2: Sales and affiliate entry age, by activity type.



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Average horizontal (export) sales shares, by affiliate first age with positive horizontal (export) sales.

Figure B.3: Export competition by sibling affiliates and horizontal sales of existing affiliates.



Notes: $t = 0$ is the year in which another affiliate of the same parent starts exporting to the market of an existing affiliate. Sample of affiliates located in Canada, the United Kingdom, and Japan. Exports are to unaffiliated parties only. OLS coefficients from regressing the log of horizontal sales for affiliate a in market $n = CAN, UK, JPN$ belonging to parent p at time $t \in [-5, 5]$ on a set of dummies indicating time from export entry of a sibling located in market $n' \neq n$ belonging to the same parent p. We include the log of the MNE global sales, the log of the US parent sales, affiliate fixed effects and year fixed effects. Standard errors are clustered at the affiliate level.

C Model's Solution

C.1 Dynamic firm's problem

The compound option structure of the model implies that it can be solved backwards. We start from the problem of a firm that already has an affiliate in country j and has to decide whether to export to any country $k \neq j$. Once this problem is solved, the value of an affiliate in j is determined. Then we can solve the problem of a firm that has to decide whether to open an affiliate in each country j. For each step of the solution, we follow the method outlined in Dixit and Pindyck (1994).

To solve the affiliate export problem, we start by solving for the value functions $V^o_{jk}(z, X, \mathbf{Q})$ and $V^e_{jk}(z, X, \mathbf{Q})$ in their continuation region. Writing the Bellman equation in (14) that describes the value of the option to export to country k for a firm with an affiliate in country j in the continuation region, taking the limit as $\Delta t \to 0$, and applying Ito's Lemma, yields the no-arbitrage condition

$$\rho V^o_{jk}(z, Y_k) = \tilde{\mu}_k Y_k {V'}^o_{jk}(z, Y_k) + \frac{\tilde{\sigma}^2_k}{2} Y^2_k {V''}^o_{jk}(z, Y_k), \tag{C.1}$$

where we acknowledge that the option value of exporting to k only depends on the realization of the composite shock Y_k. Guessing a solution for the value function and applying the method of undetermined coefficients, the value of the option of exporting to country k for an affiliate in country j has general solution given by

$$V^o_{jk}(z, Y_k) = A^o_{jk}(z) Y^{\alpha_k}_k + B^o_{jk}(z) Y^{\beta_k}_k, \tag{C.2}$$

where $\alpha_k < 0$ and $\beta_k > 1$ are the roots of $\frac{1}{2}\tilde{\sigma}^2_k \xi^2 + \left(\tilde{\mu}_k - \frac{\tilde{\sigma}^2_k}{2}\right)\xi - \rho = 0$. As $Y_k \to 0$, the option of exporting becomes worthless, so it must be that $A^o_{jk}(z) = 0$. Conversely, the option of exporting becomes more attractive as Y_k increases, so it must be that $B^o_{jk}(z) > 0$.

Similarly, writing the Bellman equation in (15) that describes the value of exporting to country k from an affiliate in country j in the continuation region, taking the limit as $\Delta t \to 0$, and applying Ito's Lemma, yields the no-arbitrage condition

$$\rho V^e_{jk}(z, Y_k) = \pi_{jk}(z, Y_k) - f^e_{jk} + \tilde{\mu}_k {V'}^e_{jk}(z, Y_k) + \frac{\tilde{\sigma}^2_k}{2} Y^2_k {V''}^e_{jk}(z, Y_k). \tag{C.3}$$

Guessing a solution for the value function and applying the method of undetermined coefficients, the value of the option of exporting to country k for an affiliate in country j has general solution given by

$$V^e_{jk}(z, Y_k) = A^e_{jk}(z)Y_k^{\alpha_k} + B^e_{jk}(z)Y_k^{\beta_k} + \frac{\pi_{jk}(z, Y_k)}{\rho - \tilde{\mu}_k} - \frac{f^e_{jk}}{\rho}. \tag{C.4}$$

Notice that, as $Y_k \to 0$, there is value from the possibility of endogenously stopping to export, so it must be that $A^e_{jk}(z) > 0$. Also, as Y_k increases, the value of exports converges to the discounted profit flow (i.e., there is no further expansion option), so it must be that $B^e_{jk}(z) = 0$.

To completely solve the affiliate export problem, we need to solve for the policy functions, which are thresholds for the realizations of the composite shock that induce the affiliate to start and stop exporting. For each country pair (j, k) and for each firm with productivity z, the parameters $B^o_{jk}(z) > 0$, $A^e_{jk}(z) > 0$, and the export entry and exit thresholds, denoted by Y^{OE}_{jk} and Y^{EO}_{jk}, respectively, can be recovered from the following system of value-matching conditions,

$$V^o_{jk}(z, Y^{OE}_{jk}) = V^e_{jk}(z, Y^{OE}_{jk}) - F^e_{jk} \quad \text{and} \quad V^o_{jk}(z, Y^{EO}_{jk}) = V^e_{jk}(z, Y^{EO}_{jk}), \tag{C.5}$$

and smooth-pasting conditions,

$$V'^o_{jk}(z, Y^{OE}_{jk}) = V'^e_{jk}(z, Y^{OE}_{jk}) \quad \text{and} \quad V'^o_{jk}(z, Y^{EO}_{jk}) = V'^e_{jk}(z, Y^{EO}_{jk}), \tag{C.6}$$

where $V'(\cdot)$ denotes the derivative of a value function with respect to the composite shock.

To determine the value of an affiliate in j, we still need to solve for the value of horizontal sales. Writing the Bellman equation in (13) that describes the value of horizontal sales for an affiliate in country j in the continuation region, taking the limit as $\Delta t \to 0$, and applying Ito's Lemma, yields the no-arbitrage condition

$$\rho V^h_j(z, Y_j) = \pi_{jj}(z, Y_j) - f^h_j + \tilde{\mu}_j Y_j V'^h_j(z, Y_j) + \frac{\tilde{\sigma}^2_k}{2} Y_j^2 V''^h_j(z, Y_j). \tag{C.7}$$

Guessing a solution for the value function and applying the method of undetermined coefficients, the value of horizontal sales for an affiliate in country j has general solution given by

$$V^h_j(z, Y_j) = A^h_j(z)Y_j^{\alpha_j} + B^h_j(z)Y_j^{\beta_j} + \frac{\pi_{jj}(z, Y_j)}{\rho - \tilde{\mu}_j} - \frac{f^h_j}{\rho}. \tag{C.8}$$

Notice that, as $Y_j \to 0$, there is value from the possibility of shutting down the affiliate, so it must be that $A^h_j(z) > 0$. As Y_j increases, the value of horizontal sales converges to the discounted profit flow, so it must be that $B^h_j(z) = 0$.

The value of an affiliate in country j, $V^a_j(z, \mathbf{Y})$ is completely characterized up to the option value parameter $A_j^h(z)$:

$$V_j^a(z,\mathbf{Y}) = A_j^h(z)Y_j^{\alpha_j} + \frac{\pi_{jj}(z,Y_j)}{\rho-\tilde{\mu}_j} - \frac{f_j^h}{\rho} + \sum_{k\in\mathcal{A}_j(z)}\left[\frac{\pi_{jk}(z,Y_k)}{\rho-\tilde{\mu}_k} - \frac{f_{jk}^e}{\rho} + A_{jk}^e(z)Y_k^{\alpha_k}\right] + \sum_{k\notin\mathcal{A}_j(z)}\left[B_{jk}^o(z)Y_k^{\beta_k}\right], \tag{C.9}$$

where $\mathcal{A}_j(z)$ denotes the set of export markets in which an affiliate of a firm with productivity z located in country j exports.

To solve the affiliate opening problem, we still need to solve for the option value of opening an affiliate, and for the policy functions. Writing the Bellman equation in (12) that describes the value of the option to open an affiliate in country j in the continuation region, taking the limit for $\Delta t \to 0$, and applying Ito's Lemma, yields the no-arbitrage condition

$$\rho V_j^o(z,Y_j) = \tilde{\mu}_j Y_j V'^o_j(z,Y_j) + \frac{\tilde{\sigma}_j^2}{2} Y_j^2 V''^o_j(z,Y_j). \tag{C.10}$$

Guessing a solution for the value function and applying the method of undetermined coefficients, the value of the option of opening an affiliate in country j has general solution given by

$$V_j^o(z,Y_j) = A_j^o(z)Y_j^{\alpha_j} + B_j^o(z)Y_j^{\beta_j}. \tag{C.11}$$

As $Y_j \to 0$, the option of opening an affiliate becomes worthless, so it must be that $A_j^o(z) = 0$. Conversely, the option of opening an affiliate becomes more attractive as Y_j increases, so it must be that $B_j^o(z) > 0$.

Let Y_j^{OH} and Y_j^{HO} denote the thresholds for the realization of the composite shock that induce a firm to open or shut down an affiliate in country j, respectively. It is important to point out that, when a firm decides to open an affiliate in a country, it considers not only the value of its horizontal sales, but also the option value of potential exports to any destination country. For this reason, the value-matching and smooth-pasting conditions that deliver the parameters $A_j^h(z)$, $B_j^o(z)$, and the policy functions Y_j^{OH} and Y_j^{HO} entail tangency conditions linking the option value function $V_j^o(z,Y_j)$ and the total value of the affiliate $V_j^a(z,\mathbf{Y})$,

$$V_j^o(z,Y_j^{OH}) = V_j^a(z,Y_j^{OH},\mathbf{Y}_{-j}) - F_j^h \quad , \quad V_j^o(z,Y_j^{HO}) = V_j^a(z,Y_j^{HO},\mathbf{Y}_{-j}), \tag{C.12}$$

$$V'^o_j(z,Y_j^{OH}) = V'^a_j(z,Y_j^{OH},\mathbf{Y}_{-j}) \quad , \quad V'^o_j(z,Y_j^{HO},\mathbf{Y}_{-j}) = V'^a_j(z,Y_{-j}^{HO}), \tag{C.13}$$

where ,$\mathbf{Y}_{-j}$ denotes the vector of composite shocks in countries other than j.

C.2 Price indexes and equilibrium wages

We now show how to solve for the vector of price indexes P_k, for $k = 1, ...N$, and for the laws of motion governing the evolution of affiliate operations over time across countries. As we only consider MNEs from the United States, we compute the price index as an aggregate of the prices associated with transactions of US MNEs and of domestic firms:

$$P_k^{1-\eta} = \lambda_{kkk} P_{kkk}^{1-\eta} + \lambda_{US,kk} P_{US,kk}^{1-\eta} + \sum_{j \neq k} \lambda_{US,jk} P_{US,jk}^{1-\eta}, \tag{C.14}$$

where P_{kkk} denotes the aggregate price of domestic varieties, $P_{US,kk}$ denotes the aggregate price of varieties produced via the horizontal operations of US affiliates in k, and $P_{US,jk}$ denotes the aggregate price of varieties produced by US affiliates in j and exported to k:

$$P_{kkk}^{1-\eta} = \int_{\Omega_{kkk}} \left(\frac{\eta}{\eta - 1} \frac{w_k}{z} \right)^{1-\eta} dG_k(z), \tag{C.15}$$

$$P_{US,kk}^{1-\eta} = \int_{\Omega_{US,kk}} \left(\frac{\eta}{\eta - 1} \frac{w_k}{zZ} \right)^{1-\eta} dG_{US}(z), \tag{C.16}$$

$$P_{US,jk}^{1-\eta} = \int_{\Omega_{US,jk}} \left(\frac{\eta}{\eta - 1} \frac{\tau_{jk} w_j}{zZ} \right)^{1-\eta} dG_{US}(z) \tag{C.17}$$

and Ω_{kkk}, $\Omega_{US,kk}$, $\Omega_{US,jk}$, denote the corresponding sets of varieties produced. $G_k(z)$ denotes the exogenous distribution of productivity of firms from country k.

The price indexes depend directly on the US productivity shock $Z = e^X$ and indirectly —via the integration sets— on the demand shocks Q_k. Moreover, as shown below, the integration sets themselves depend on the entry and exit thresholds, which in turn depend on the price indexes. To solve the aggregation problem, we appeal to the equivalence result shown in Leahy (1993): when solving the entry and exit problem, each firm takes aggregate prices and the sets of firms operating in each country as given, and does not take into account the effect of its own entry and exit decisions on these variables.

We assume that the mass of firms in each country k, M_k, is constant. The endogenous mass of affiliates of US firms located in j, $M_{US,j}$, is given by continuing plus new affiliates,

$$M'_{US,j} = M_{US,j} \cdot (1 - G_{US}(z_{US,j}^{HO})) + (M_{US} - M_{US,j}) \cdot (1 - G_{US}(z_{US,j}^{OH})), \tag{C.18}$$

where $z_{US,j}^{OH}$ ($z_{US,j}^{HO}$) is the productivity threshold that induces a US firm to open (shut down) an affiliate in j. Notice that $z_{US,j}^{OH}(Y_j)$ ($z_{US,j}^{HO}(Y_j)$) is the inverse of the threshold in the realization of the shock $Y_j^{OH}(z)$ ($Y_j^{HO}(z)$), whose existence is guaranteed by the monotonicity property of the thresholds shown in Proposition 1.

Similarly, the mass of affiliates of US firms located in j that export to k is given by continuing plus new exporters to k,

$$M'_{US,jk} = M_{US,jk} \cdot (1 - G_{US}(z^{EO}_{US,jk})) + (M_{US,j} - M_{US,jk}) \cdot (1 - G_{US}(z^{OE}_{US,jk})), \quad \text{(C.19)}$$

where $z^{OE}_{US,jk}$ ($z^{EO}_{US,jk}$) is the productivity threshold that induces an affiliate of a US firm in j to start (stop) exporting to k.

Since we take Q_k as exogenous, we can simply backup L_k from the labor market aggregation conditions,

$$L_k = L_{kkk} + L_{US,kk} + \sum_{j \neq k} L_{US,kj}$$

where L_k denotes labor force in country k, L_{kkk} denotes the labor offered by workers in country k working for firms from country k, and $L_{US,kk}$ and $L_{US,kj}$ denote the labor offered by workers in country k working for horizontal and exporting affiliates of US MNEs, respectively. The domestic labor aggregate can be computed as

$$L_{kkk}(t) = \int_{\Omega_{kkk}(t)} q_{kkk}(t)/z dz = \int_{\Omega_{kkk}(t)} p_{kkk}^{-\eta} P_{kkk} Q_{kkk}/z dz.$$

Notice that since foreign productivity does not change, this should be constant. The labor aggregates for US MNEs can be computed as

$$L_{US,kk}(t) = \int_{\Omega_{US,kk}(t)} q_{US,kk}(t)/(zZ(t))dz = \int_{\Omega_{US,kk}(t)} p_{US,kk}(t)^{-\eta} P_{US,kk}(t) Q_{US,kk}(t)/(zZ(t))dz,$$
$$L_{US,kj}(t) = \int_{\Omega_{US,kj}(t)} q_{US,kj}(t)/(zZ(t))dz = \int_{\Omega_{US,kj}(t)} p_{US,kj}(t)^{-\eta} P_{US,kj}(t) Q_{US,kj}(t)/(zZ(t))dz.$$

Wages are pinned down from (22).

C.3 Proofs

In this section, we provide proofs for Propositions 1 and 2. To this end, notice that, when $A^e_j(z) = A^e_{jk}(z) = 0$, the systems of value-matching and smooth pasting conditions (C.5)-(C.6) and (C.12)-

(C.13) only identify the entry thresholds Y_j^{OH}, Y_{jk}^{OE} and can be solved in closed form, delivering the expressions in (23) and (24). Additionally, the option value parameters $B_j^o(z)$, $B_{jk}^o(z)$ can be characterized in closed form,

$$B_j^o(z) = \beta_j^{-\beta_j}(\beta_j-1)^{\beta_j-1}\cdot\left(\frac{f_j^h+\rho F_j^h}{\rho}-\mathbf{V}_j^E(z,\mathbf{Y}_{-j})\right)^{1-\beta_j}\cdot\left(\frac{\kappa_{jj}(z)}{\rho-\tilde{\mu}_j}\right)^{\beta_j}, \tag{C.20}$$

$$B_{jk}^o(z) = \beta_k^{-\beta_k}(\beta_k-1)^{\beta_k-1}\cdot\left(\frac{f_{jk}^e+\rho F_{jk}^e}{\rho}\right)^{1-\beta_k}\cdot\left(\frac{\kappa_{jk}(z)}{\rho-\tilde{\mu}_k}\right)^{\beta_k}, \tag{C.21}$$

where $\kappa_{jk}(z)\equiv H(\tau_{jk}w_j/z)^{1-\eta}P_k^\eta\lambda_{jk}$ and $\mathbf{V}_j^E(z,\mathbf{Y}_{-j})=\sum_{k\in\mathcal{A}_j(z)}\left[\frac{\kappa_{jk}(z)Y_k}{\rho-\tilde{\mu}_k}-\frac{f_{jk}^e}{\rho}\right]+\sum_{k\notin\mathcal{A}_j(z)}\left[B_{jk}^o(z)Y_k^{\beta_k}\right]$.

Equations (C.20) and (C.21) reveal that the option value of opening an affiliate (exporting from an affiliate) is decreasing in both the fixed and sunk costs of affiliate opening (exporting). In addition, the option value of opening an affiliate is increasing in the value of the potential export network of the affiliate, highlighting the effects of the compound option mechanism. Finally, both the option values of opening and exporting from an affiliate are increasing in firm productivity z.

Proof of Proposition 1. Affiliate's variable profits and the value of an affiliate's export network are increasing in firm productivity,

$$\frac{\partial\kappa_{jk}(z)}{\partial z} > 0, \tag{C.22}$$

$$\frac{\partial\mathbf{V}_j^E(z,\mathbf{Y}_{-j})}{\partial z} > 0. \tag{C.23}$$

Given (C.22) and (C.23), the proof for the affiliate export entry threshold is immediate from taking derivatives in (24),

$$\frac{\partial Y_{jk}^{OE}(z)}{\partial z} = \underbrace{\left(\frac{\beta_k}{\beta_k-1}\right)\cdot\left(\frac{f_{jk}^e+\rho F_{jk}^e}{\rho}\right)\cdot(\rho-\tilde{\mu}_k)}_{\geq 0}\cdot\underbrace{\left(\frac{1}{-\kappa_{jk}(z)^2}\right)}_{<0}\cdot\underbrace{\frac{\partial\kappa_{jk}(z)}{\partial z}}_{>0}\leq 0.$$

We compute the derivative in the scenario in which $(f_j^h+\rho F_j^h)/\rho-\mathbf{V}_j^E(z,\mathbf{Y}_{-j})>0$, so that $Y_j^{OH}(z)>0$ and the entry problem is well-defined. Taking derivative in (24) yields

$$\frac{\partial Y_j^{OH}(z)}{\partial z} = \underbrace{\left(\frac{\beta_j}{\beta_j-1}\right)}_{>0}\cdot\left\{\underbrace{\frac{-\partial\mathbf{V}_j^E(z,\mathbf{Y}_{-j})}{\partial z}\cdot\frac{\rho-\tilde{\mu}_j}{\kappa_{jj}(z)}}_{<0}+\underbrace{\left(\frac{f_j^h+\rho F_j^h}{\rho}-\mathbf{V}_j^E(z,\mathbf{Y}_{-j})\right)}_{\geq 0}\cdot\underbrace{\left(\frac{\rho-\tilde{\mu}_j}{-\kappa_{jj}(z)^2}\right)\cdot\frac{\partial\kappa_{jj}(z)}{\partial z}}_{<0}\right\}\leq 0.$$

Corollary 1. *More productive firms enter more export markets.*

Proof. $\frac{\partial\left[prob\left\{y\geq Y_j^{OH}(z)\right\},\forall j\right]}{\partial z} = \frac{\partial\left[prob\left\{y\geq Y_j^{OH}(z)\right\},\forall j\right]}{\partial Y_j^{OH}(z)} \cdot \frac{\partial Y_j^{OH}(z)}{\partial z}$ where the first terms is negative and Proposition 1 implies that the second term is weakly negative, so $\frac{\partial\left[prob\left\{y\geq Y_j^{OH}(z)\right\},\forall j\right]}{\partial z} \geq 0$.

Corollary 2. *The mass of firms having affiliates in n host markets is decreasing in n.*

Proof. Without loss of generality, let us assume that all firms only sell in the US and are considering whether and where to open affiliates ($t = 1$). Let us order the productivity thresholds needed to open an affiliate in a market in ascending order, and let $\bar{Z}_n^{OH}$ ($\bar{Z}_{n-1}^{OH}$) denote the maximum among the first n ($n-1$) productivity thresholds, so that $\bar{Z}_n^{OH} \geq \bar{Z}_{n-1}^{OH}$. Let M_j denote the mass of firms that at $t = 1$ open affiliates in country j: $M_j = \int_{Z_j^{OH}}^{\infty} dG(z)$, so $\frac{\partial M_j}{\partial Z_j^{OH}} \leq 0$. Let $\bar{M}_n$ ($\bar{M}_{n-1}$) denote the mass of firms that at $t = 1$ open affiliates in the n ($n-1$) countries with the lowest productivity thresholds. Then $\bar{Z}_n^{OH} \geq \bar{Z}_{n-1}^{OH}$ and $\frac{\partial M_j}{\partial Z_j^{OH}} \leq 0$ implies that $\bar{M}_n \leq \bar{M}_{n-1}, \forall n$.

Proof of Proposition 2. Affiliate's variable profits are increasing in the taste shifter,

$$\frac{\partial \kappa_{jj}(z)}{\partial \lambda_j} \quad > \quad 0. \tag{C.24}$$

We compute the derivative in the scenario in which $(f_j^h + \rho F_j^h)/\rho - \mathbf{V}_j^E(z, \mathbf{Y}_{-j}) > 0$, so that $Y_j^{OH}(z) > 0$ and the entry problem is well-defined. Given (C.24), the proof is immediate from taking derivatives in (23),

$$\frac{\partial Y_j^{OH}(z)}{\partial \lambda_j} = \underbrace{\frac{\beta_j}{\beta_j - 1} \cdot \left(\frac{f_j^h + \rho F_j^h}{\rho} - \mathbf{V}_j^E(z, \mathbf{Y}_{-j})\right)}_{\geq 0} \cdot \underbrace{\left(\frac{\rho - \tilde{\mu}_j}{-\kappa_{jj}(z)^2}\right)}_{<0} \cdot \underbrace{\frac{\partial \kappa_{jj}(z)}{\partial \lambda_j}}_{>0} \leq 0.$$

Additionally, taking derivatives with respect to F_j^h in (23) yields

$$\frac{\partial Y_j^{OH}(z)}{\partial F_j^h} = \frac{\beta_j}{\beta_j - 1} \cdot \left(\frac{\rho - \tilde{\mu}_j}{\kappa_{jj}(z)}\right) > 0.$$

D Calibration: Additional Results

Table D.1: Calibrated parameters: shock processes and wages.

	$\tilde{\mu}_j$	$\tilde{\sigma}_j$	γ_j	$Q_j(0)$	w_j
Brazil	0.051	0.130	0.032	0.096	0.711
Canada	0.030	0.136	0.661	0.073	0.831
China	0.064	0.122	0.035	1.162	0.307
France	0.025	0.127	0.370	0.125	0.981
United Kingdom	0.025	0.136	0.314	0.128	0.930
Germany	0.026	0.128	0.648	0.222	0.825
Ireland	0.048	0.144	0.560	0.003	1.188
Japan	0.027	0.133	0.383	0.354	0.719
Mexico	0.036	0.127	0.083	0.078	0.699
Singapore	0.080	0.137	0.213	0.003	0.950
United States	0.028	0.116	1.000	1.000	1.000

Note: $\tilde{\mu}_j$ and $\tilde{\sigma}_j$ refer to the drift and standard deviation, respectively, of the composite shock Y_j. γ_j refers to the correlation between the US aggregate productivity shock and country j's aggregate demand shock. $Q_j(0)$ denotes the initial value of the demand shock process in country j. w_j refers to country j's wage relative to the US.

Table D.2: Calibrated parameters: fixed costs, sunk costs, and taste shifters.

	f_j^h	F_j^h	$f_{j,US}^e$	$F_{j,US}^e$	f_{jk}^e	F_{jk}^e	λ_j
Brazil	0.0113	0.0023	0.0028	0.0000	0.0023	0.00002	0.051
Canada	0.0121	0.0002	0.0040	0.0008	0.0024	0.0000	0.161
China	0.0016	0.0000	0.0012	0.0000	0.0009	0.0000	0.013
France	0.0165	0.0007	0.0023	0.0000	0.0053	0.0022	0.097
United Kingdom	0.0122	0.0000	0.0023	0.0003	0.0044	0.0003	0.104
Germany	0.0167	0.0004	0.0025	0.0001	0.0060	0.0024	0.088
Ireland	0.0027	0.0003	0.0015	0.0000	0.0014	0.0007	0.010
Japan	0.0327	0.0024	0.0042	0.0000	0.0051	0.0001	0.097
Mexico	0.0047	0.0001	0.0014	0.0001	0.0010	0.00001	0.020
Singapore	0.0029	0.00002	0.0033	0.0002	0.0028	0.0009	0.013

Note: f_j^h (F_j^h) is the fixed (sunk) cost of opening an affiliate in country j. $f_{j,US}^e$ ($F_{j,US}^e$) is the fixed (sunk) cost of exporting from j to the United States. f_{jk}^e (F_{jk}^e) is the fixed (sunk) cost of exporting from j to a destination k other than the United States. λ_j is the taste shifter associated with goods produced by US MNEs in country j.

Table D.3: Calibrated parameters: bilateral iceberg-trade costs.

	BRA	CAN	CHN	FRA	GBR	GER	IRL	JPN	MEX	SGP	USA
BRA	1.000	2.579	1.000	2.470	3.467	2.419	2.360	2.551	1.825	2.684	5.415
CAN	1.675	1.000	1.000	1.828	2.516	1.780	1.684	1.821	1.221	2.204	3.230
CHN	7.933	9.552	1.000	7.663	11.446	7.301	7.338	6.313	6.747	6.125	16.55
FRA	2.265	1.773	1.000	1.000	1.409	1.138	1.311	1.544	2.102	2.625	3.638
GBR	2.569	1.620	1.000	1.244	1.000	1.328	1.286	1.375	2.359	2.976	3.369
GER	2.829	2.199	1.059	1.450	1.573	1.000	1.691	1.815	2.608	3.205	4.090
IRL	3.404	2.009	1.312	2.060	1.737	2.085	1.000	1.745	3.090	3.989	4.120
JPN	2.174	2.565	1.000	2.166	2.984	2.070	2.062	1.000	1.776	1.733	5.236
MEX	2.411	3.171	1.105	3.030	3.659	2.946	2.832	2.753	1.000	3.345	5.685
SGP	1.863	2.170	1.000	1.985	2.093	1.903	1.919	1.545	1.758	1.000	4.145
USA	2.359	2.481	1.264	2.002	2.057	2.079	2.231	2.162	2.056	2.578	1.000

Note: Trade costs from country j (rows) to destination k (columns).

Table D.4: Static moments: intensive margin. Model versus data.

Share of:	Affiliate sales to host market		Affiliate sales to the US		Affiliate sales to third countries	
	data	model	data	model	data	model
Brazil	0.018	0.018	0.072	0.072	0.142	0.140
Canada	0.048	0.048	0.261	0.261	0.113	0.111
China	0.003	0.003	0.099	0.099	0.219	0.219
France	0.038	0.039	0.075	0.075	0.364	0.364
United Kingdom	0.052	0.053	0.111	0.111	0.371	0.366
Germany	0.047	0.048	0.092	0.092	0.413	0.413
Ireland	0.002	0.002	0.242	0.242	0.515	0.605
Japan	0.034	0.034	0.047	0.047	0.130	0.130
Mexico	0.011	0.011	0.173	0.173	0.128	0.128
Singapore	0.003	0.003	0.222	0.222	0.488	0.488
Average	0.026	0.026	0.139	0.139	0.288	0.296

Note: Affiliate sales to the host market are expressed as a share of the parent's US sales. Affiliate sales to the US and to third countries are expressed as a share of affiliate sales in the host market. Calculations are conditional on affiliate entry, but unconditional on affiliate exports. Averages across years.

Table D.5: Static moments: intensive margin. Selected destinations. Model versus data.

Share of:	Affiliate sales to Canada		Affiliate sales to the United Kingdom		Affiliate sales to Japan	
	data	model	data	model	data	model
Brazil	0.008	0.015	0.008	0.005	0.004	0.005
Canada	n.a.	n.a.	0.006	0.005	0.003	0.005
China	0.008	0.004	0.002	0.003	0.037	0.046
France	0.010	0.006	0.091	0.122	0.006	0.005
United Kingdom	0.012	0.011	n.a.	n.a.	0.009	0.010
Germany	0.009	0.003	0.079	0.136	0.010	0.003
Ireland	0.053	0.042	0.386	0.395	0.082	0.037
Japan	0.006	0.006	0.001	0.005	n.a.	n.a.
Mexico	0.007	0.011	0.001	0.010	0.002	0.009
Singapore	0.024	0.024	0.047	0.106	0.147	0.116
Average	0.015	0.014	0.069	0.087	0.033	0.026

Note: Affiliate sales to destination j are expressed as share of sales in the affiliate host market. Calculations are conditional on affiliate entry, but unconditional on affiliate exports. Averages across (benchmark) years.

Table D.6: Static moments: extensive margin. Model versus data.

Share of:	MNEs with affiliates in j		Affiliates in j exporting to the US		Affiliates in j exporting to third countries	
	data	model	data	model	data	model
Brazil	0.198	0.189	0.515	0.515	0.674	0.668
Canada	0.544	0.539	0.725	0.722	0.478	0.478
China	0.184	0.181	0.382	0.382	0.548	0.545
France	0.312	0.310	0.539	0.542	0.747	0.746
United Kingdom	0.554	0.560	0.605	0.605	0.739	0.748
Germany	0.367	0.364	0.608	0.608	0.760	0.760
Ireland	0.122	0.122	0.575	0.574	0.760	0.751
Japan	0.155	0.150	0.468	0.468	0.578	0.541
Mexico	0.302	0.305	0.647	0.649	0.494	0.498
Singapore	0.129	0.113	0.597	0.596	0.724	0.721
Average	0.287	0.283	0.566	0.566	0.650	0.646

Note: MNEs with affiliates in j are expressed as shares of the total number of US MNEs. Exporting affiliates are expressed as shares of the total number of affiliates in j. Calculations are conditional on affiliate entry. Averages across years.

Table D.7: Dynamic moments: entry. Model versus data.

Share of:	MNEs opening affiliates in j		Affiliates in j that start exporting to:			
			the United States		third countries	
	data	model	data	model	data	model
Brazil	0.021	0.012	0.032	0.023	0.037	0.028
Canada	0.060	0.031	0.024	0.018	0.036	0.028
China	0.029	0.017	0.027	0.019	0.040	0.031
France	0.038	0.022	0.033	0.032	0.025	0.020
United Kingdom	0.069	0.048	0.029	0.021	0.027	0.027
Germany	0.046	0.026	0.030	0.030	0.025	0.019
Ireland	0.015	0.006	0.037	0.030	0.030	0.043
Japan	0.020	0.014	0.032	0.023	0.030	0.028
Mexico	0.036	0.024	0.029	0.025	0.033	0.022
Singapore	0.019	0.005	0.026	0.019	0.028	0.022
Average	0.035	0.021	0.030	0.024	0.031	0.027

Note: MNEs opening affiliates in j are expressed as shares of the total number of US MNEs in the period before entry. Affiliates that start exporting are expressed as shares of the total number of affiliates in j in the period before entry. Calculations are conditional on affiliate entry. Averages across years.

Table D.8: Dynamic moments: exit. Model versus data.

Share of:	MNEs shutting down affiliates in j		Affiliates in j that stop exporting to			
			the United States		third countries	
	data	model	data	model	data	model
Brazil	0.102	0.074	0.027	0.043	0.032	0.037
Canada	0.125	0.083	0.021	0.022	0.028	0.058
China	0.082	0.105	0.020	0.054	0.031	0.051
France	0.117	0.088	0.029	0.058	0.021	0.018
United Kingdom	0.128	0.126	0.026	0.031	0.023	0.037
Germany	0.122	0.088	0.029	0.048	0.024	0.017
Ireland	0.119	0.046	0.030	0.055	0.030	0.053
Japan	0.111	0.084	0.029	0.048	0.029	0.046
Mexico	0.114	0.088	0.021	0.032	0.026	0.038
Singapore	0.115	0.044	0.023	0.029	0.026	0.014
Average	0.113	0.083	0.025	0.042	0.027	0.037

Note: MNEs shutting down affiliates in j are expressed as shares of the total number of affiliates in j in the period before exit. Affiliates that stop exporting are expressed as shares of the total number of affiliates in j that export in the period before entry. Calculations are conditional on affiliate entry. Averages across years.

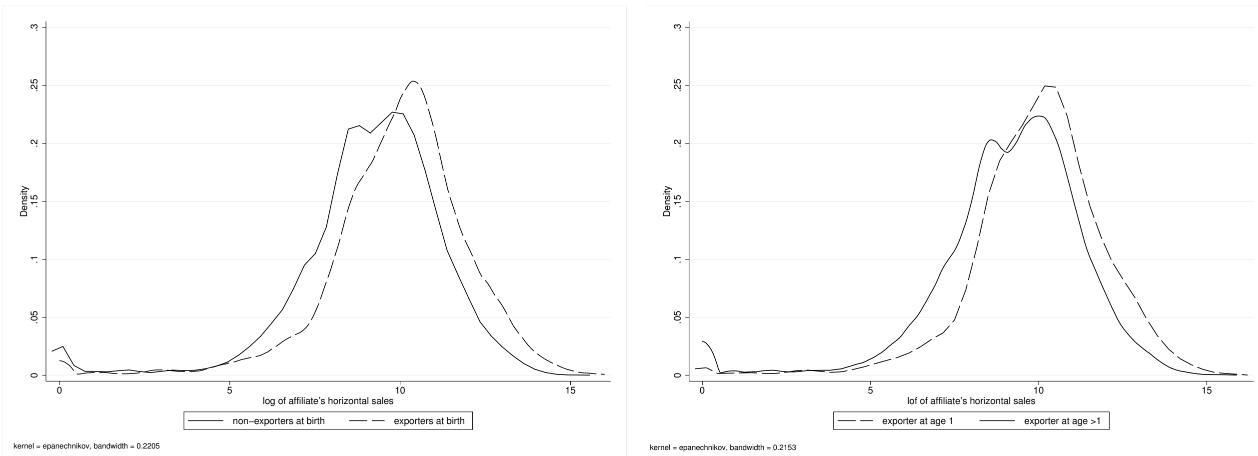
Table D.9: Exporter and early-exporter size advantage. Model versus data.

	Exporter size advantage		Early exporter size advantage	
	data	model	data	model
Brazil	6.31	7.43	3.49	5.55
Canada	3.39	6.81	2.52	5.56
China	7.84	7.57	3.08	5.71
France	4.46	7.08	1.95	5.81
United Kingdom	1.93	6.91	1.52	5.56
Germany	5.47	6.97	4.24	5.56
Ireland	8.02	6.09	8.57	5.22
Japan	12.49	7.06	2.31	5.39
Mexico	4.21	7.00	2.08	5.67
Singapore	8.59	6.78	7.04	5.38
Average	6.27	6.97	3.68	5.54

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Exporter size advantage refers to the average size of exporting MNE affiliates relative to the average size of non-exporting MNE affiliates, an average across countries and years. Early-exporter size advantage refers to the average size of exporting MNE affiliates that start exports early in life relative to the average size of exporting MNE affiliates that start exports later in life. Size refers to horizontal affiliate sales; early versus late exporters refers to affiliates that are born with exports versus the ones that start exporting later. Calculations in the calibrated model trim the upper and lower 10th decile of the simulated firm-level data.

Figure D.1: Exporter and early-exporter size advantage, OLS.

(a) Exporters vs non-exporters (b) Early vs late exporters



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Kernel density of log horizontal sales for affiliates that: are born with exclusively horizontal sales (non-exporters) and those with exports (exporters), in (D.1a); start exporting in their first year of life and those that start after their first year of life, in (D.1b).

Table D.10: First-affiliate size and cost advantage, OLS.

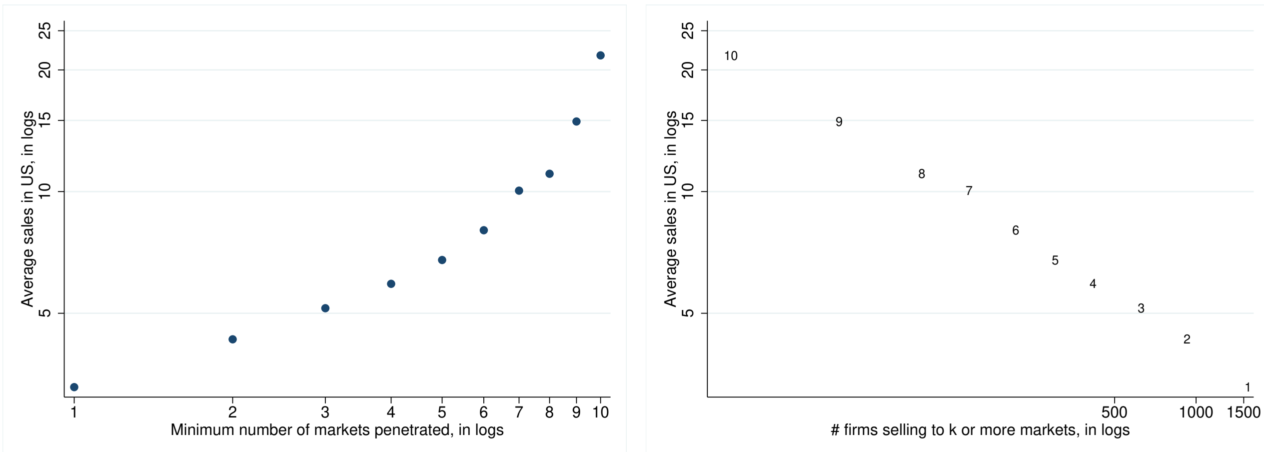
Dependent variable	probability of being the first affiliate of a MNE			
	(1)	(2)	(3)	(4)
Log of horizontal sales	0.013***	0.012***	0.012***	0.011***
	(0.003)	(0.003)	(0.003)	(0.003)
Number of business procedures		-0.006***		-0.008***
		(0.001)		(0.002)
Cost of starting business			-0.006***	-0.0004*
(% of GDP p.c.)			(0.0002)	(0.0002)
Obs	36,127	36,127	36,127	36,127
R-squared	0.291	0.295	0.293	0.297

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. The variables "Number of business procedures" and "Cost of starting a business" are from the World Bank's Doing Business dataset. All specifications include host country GDP (from Penn World Tables, 9.0), year fixed effects, and parent fixed effects. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted $^{***}p < 0.01$, $^{**}p < 0.05$, and $^{*}p < 0.1$.

Figure D.2: Parent size in the United States and MNE entry

(a) Number of markets penetrated

(b) Number of firms penetrating multiple markets



Notes: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Slopes are: (D.2a) 0.736 (s.e. 0.121); (D.2b) -0.424 (s.e. 0.008).

E Quantitative Analysis: Additional Results

Table D.11: Calibrated MNE costs, as share of sales, by country.

	Sunk costs F_j^h (% of parent US sales)			Fixed costs f_j^h (% of horizontal sales)		
Sales percentiles	5th	50th	95th	5th	50th	95th
Brazil	0.189	0.068	0.007	16.25	9.960	1.530
Canada	0.023	0.014	0.002	22.74	14.36	2.402
China	2.e-04	1.e-04	0.000	11.82	7.013	2.778
France	0.075	0.035	0.005	22.46	15.81	2.547
United Kingdom	2.e-04	1.e-04	0.000	23.93	14.90	2.386
Germany	0.037	0.021	0.003	22.60	15.52	2.483
Ireland	0.009	0.006	0.001	18.92	10.85	1.822
Japan	0.163	0.066	0.009	22.63	16.30	2.388
Mexico	0.010	0.003	4.e-04	17.86	12.68	2.269
Singapore	0.001	3.e-04	0.000	7.740	4.432	2.849
Average	0.051	0.021	0.003	18.69	12.18	2.345

Note: Sales evaluated at the year of affiliate entry.

Table D.12: Calibrated MNE export costs, as share of sales, by country.

	Sunk export costs F^e_{jk} (% of horizontal affiliate sales)						Fixed export costs f^e_{jk} (% of average affiliate exports)					
	to United States			to other countries			to United States			to other countries		
Sales percentiles	5th	50th	95th	5th	50th	95th	5th	50th	95th	5th	50th	95th
Brazil	0.009	0.003	0.001	0.049	0.0172	0.002	11.89	9.569	1.695	12.07	11.85	3.365
Canada	1.499	0.765	0.121	0.000	0.000	0.000	11.84	9.822	1.604	14.45	12.52	3.657
China	0.027	0.013	0.003	0.020	0.008	0.001	12.96	9.511	1.751	12.67	11.39	3.371
France	0.008	0.003	4e-04	4.7667	1.667	0.206	12.13	9.901	1.466	12.17	11.69	3.024
United Kingdom	0.423	0.203	0.029	0.660	0.245	0.036	12.06	10.11	1.614	12.74	10.31	3.708
Germany	0.139	0.046	0.006	6.587	1.782	0.252	12.88	10.09	1.433	12.35	11.75	2.214
Ireland	0.000	0.000	0.000	11.28	3.706	0.634	13.92	10.88	1.894	11.68	10.01	3.527
Japan	0.002	4.e-04	1.e-04	0.102	0.019	0.003	13.71	11.09	1.99	13.74	13.14	4.12
Mexico	0.596	0.222	0.029	0.031	0.011	0.002	12.95	10.60	1.493	14.03	11.35	2.892
Singapore	0.580	0.328	0.086	6.059	2.404	0.705	12.16	9.653	1.708	12.39	12.02	3.207
Average	0.328	0.158	0.028	2.956	0.986	0.184	12.65	10.13	1.666	12.83	11.60	3.309

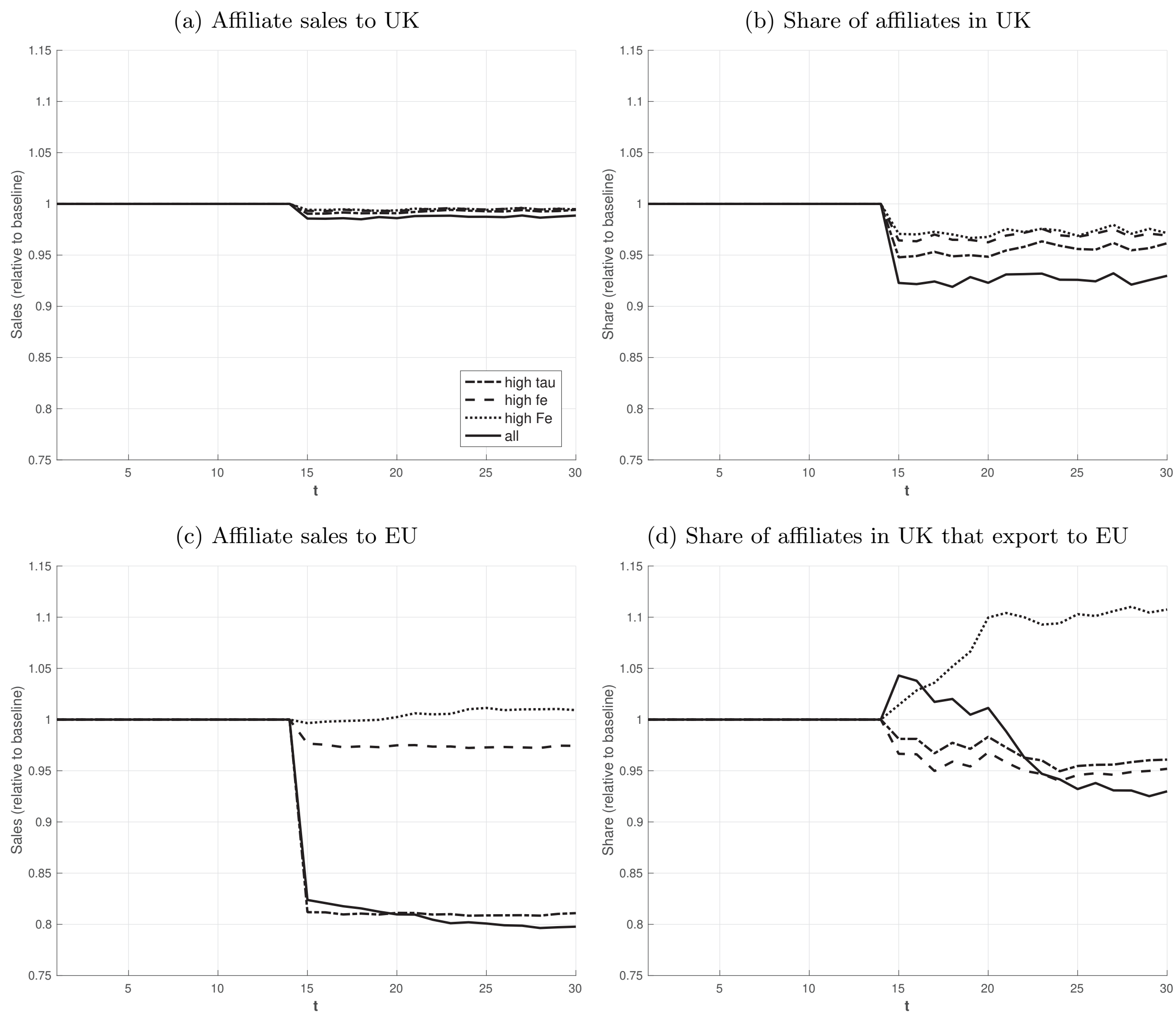
Note: Sales evaluated at the year of first export.

Table D.13: MNE sales distribution, in thousands of dollars, by country. BEA data.

	Parent sales	Affiliate horizontal sales		Affiliate export sales, avg across years	
	at age 1 of the affiliate	avg across years	at age of first export	total	to US
Brazil	1,194,252	17,013	14,404	2,814	576
Canada	475,426	14,085	12,871	4,916	2,453
China	887,290	10,329	5,024	5,025	738
France	709,151	17,443	14,468	7,586	622
Germany	657,868	19,761	15,358	12,370	937
United Kingdom	442,758	12,505	10,202	6,203	666
Ireland	974,741	5,892	4,731	16,317	1,371
Japan	575,766	15,017	11,187	2,058	374
Mexico	757,420	8,042	6,486	4,221	1,285
Singapore	859,366	5,097	3,927	8,692	705
Average	753,404	12,518	9,866	7,020	973

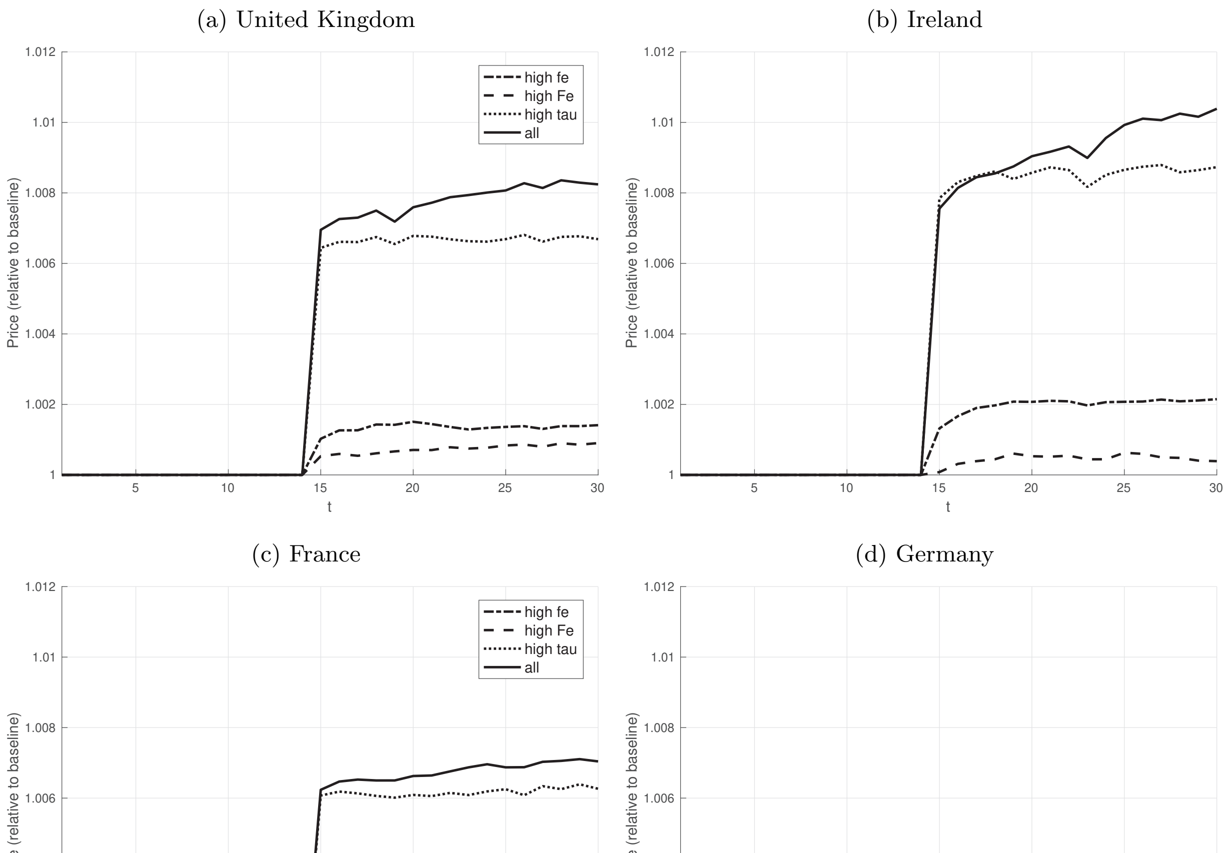
Note: Sales refer to the median firm and are in thousands of US dollars. For confidentiality purposes, magnitudes are averages across the nine observations around the median.

Figure E.1: Brexit: US affiliates in the United Kingdom. Exogenous price indexes.



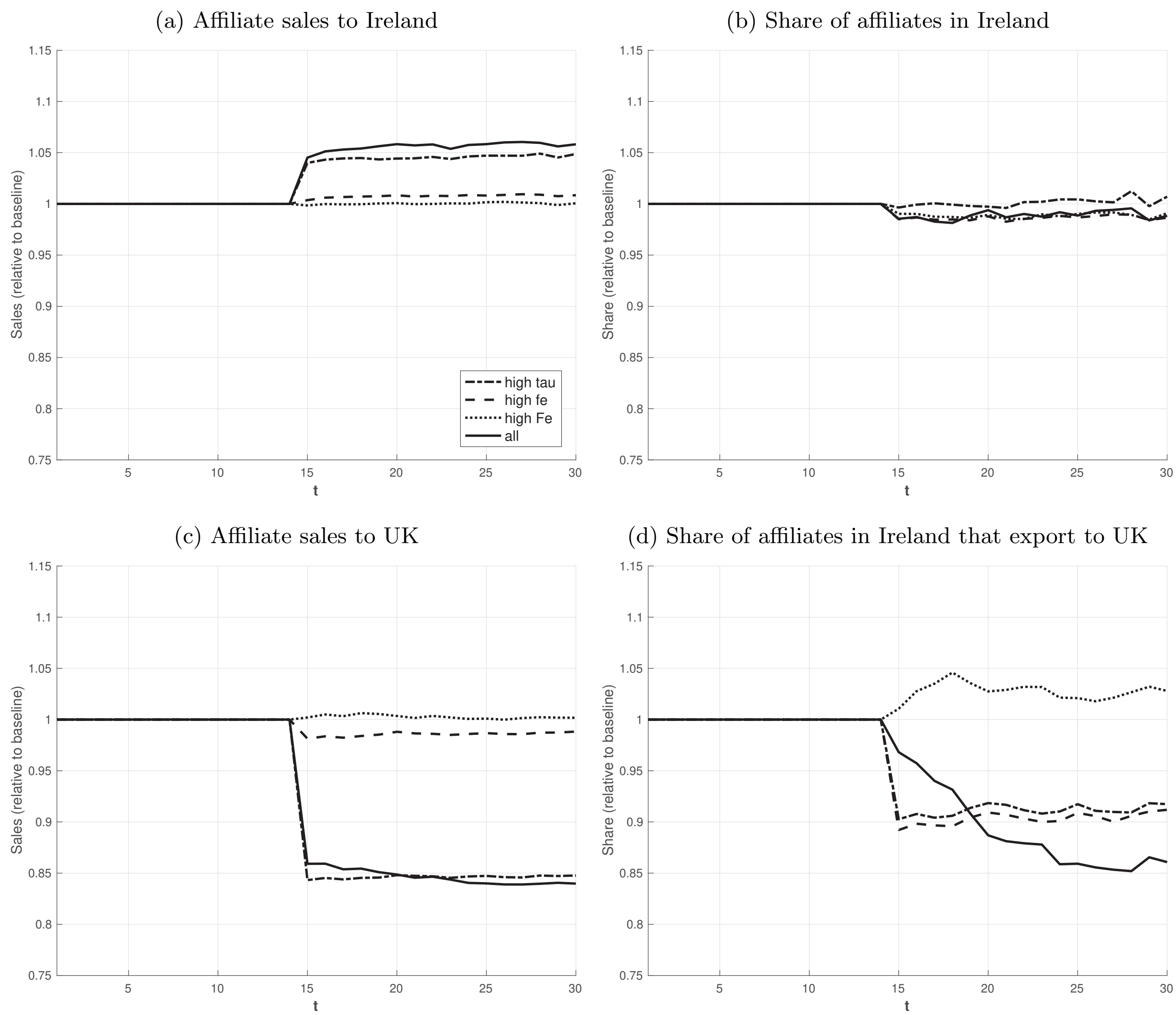
Note: "high X" refers to an increase in the barrier X from/to United Kingdom to/from country j ("shallow Brexit"). "All" refers to increasing all three export barriers from/to the United Kingdom to/from j at once ("deep Brexit"). Country j refers to Ireland, Germany, and France.

Figure E.2: Brexit: Price changes.



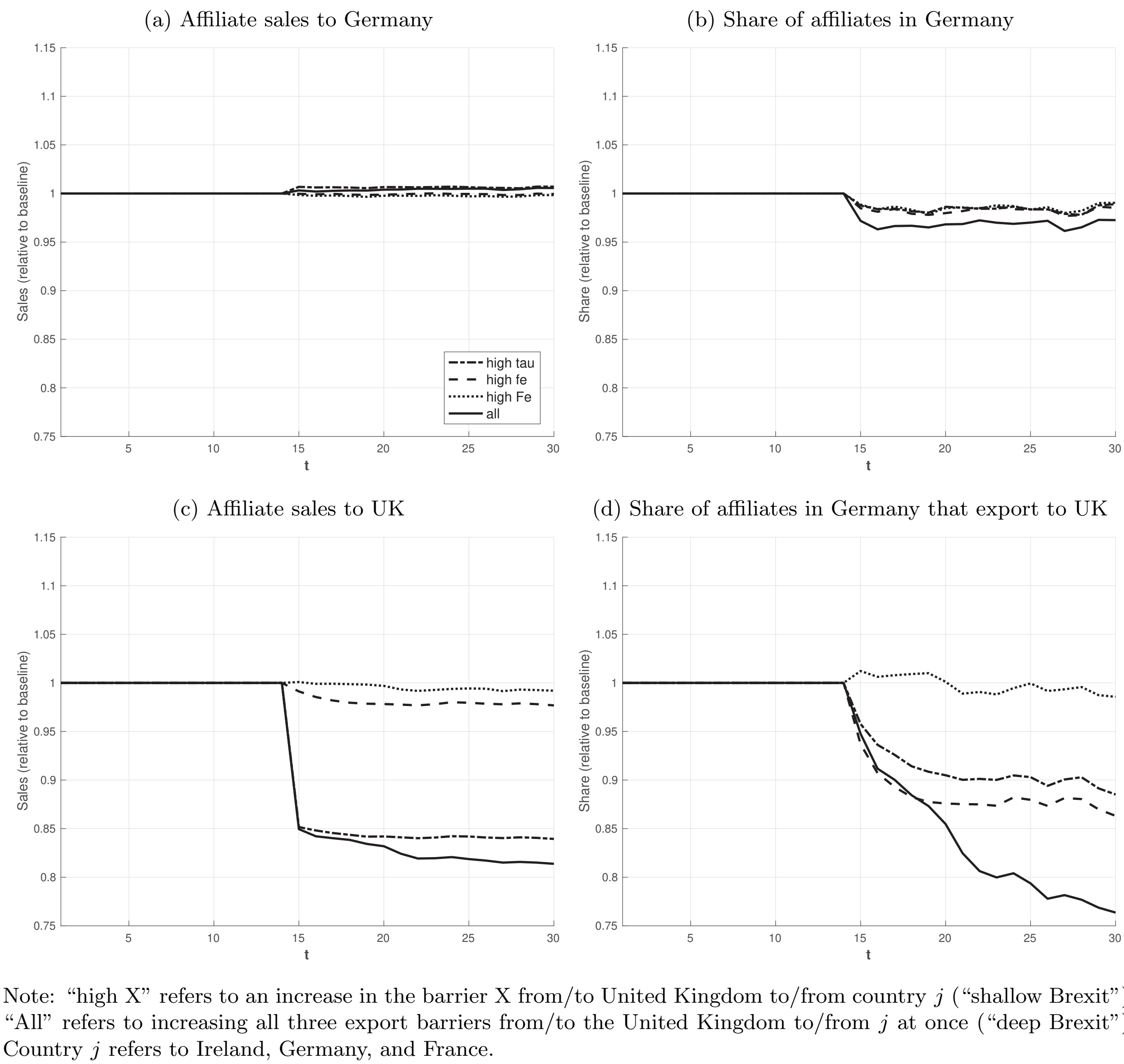
Note: “high X” refers to an increase in the barrier X from/to United Kingdom to/from country j (“shallow Brexit”). “All” refers to increasing all three export barriers from/to the United Kingdom to/from j at once (“deep Brexit”). Country j refers to Ireland, Germany, and France.

Figure E.3: Brexit: US affiliates in Ireland.



Note: “high X” refers to an increase in the barrier X from/to United Kingdom to/from country j (“shallow Brexit”). “All” refers to increasing all three export barriers from/to the United Kingdom to/from j at once (“deep Brexit”). Country j refers to Ireland, Germany, and France.

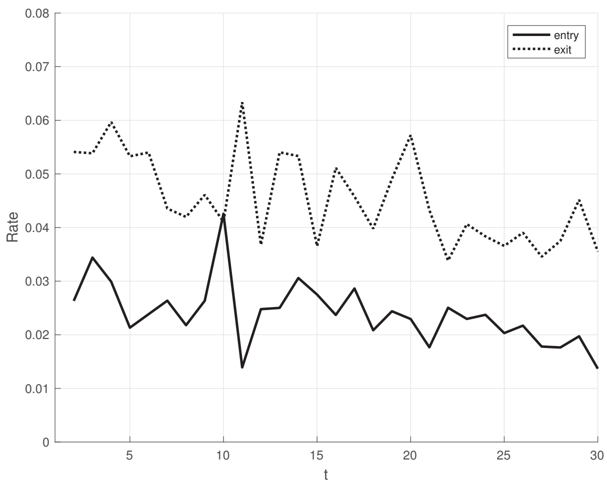
Figure E.4: Brexit: US affiliates in Germany.



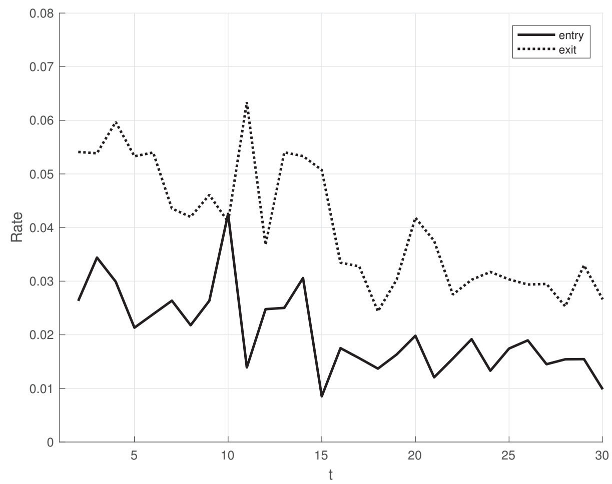
Note: "high X" refers to an increase in the barrier X from/to United Kingdom to/from country j ("shallow Brexit"). "All" refers to increasing all three export barriers from/to the United Kingdom to/from j at once ("deep Brexit"). Country j refers to Ireland, Germany, and France.

Figure E.5: Brexit: Entry and exit from the United Kingdom into EU.

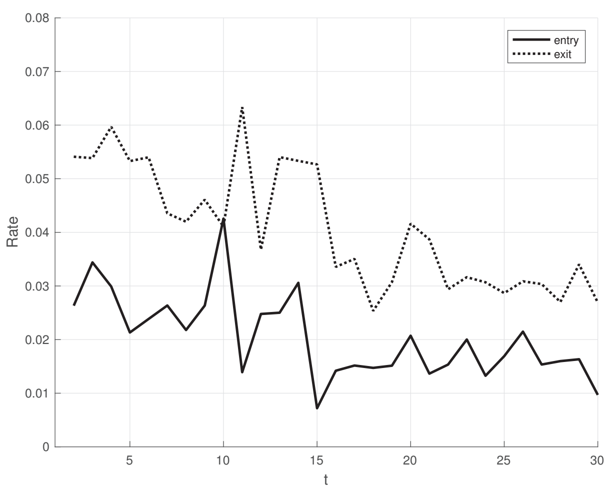
(a) Baseline



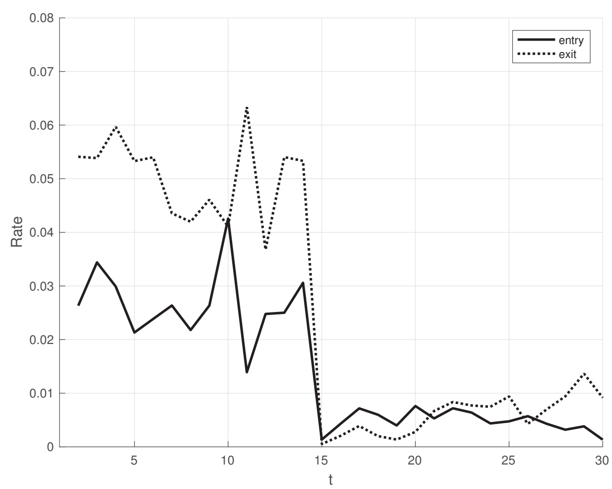
(b) High iceberg trade costs, τ



(c) High fixed export costs, f^e



(d) High sunk export costs, F^e



Note: Price indexes are endogenous.