Mean-Semivariance Portfolio Optimization and Asset Skewness

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Abstract

Markowitz style mean-variance portfolio (MVO) optimization is widely used both in academia and among financial practitioners. Its limitation, on the other hand, is also obvious: it treats downside risks indistinguishably from upside risks. Semivariance is thus proposed to substitute for variance in the optimization process. However, literature on the relative performance between MVO and mean-semivariance optimization (MSO) is still lacking. This paper studies the performance of MVO and MSO under varying asset return distributions through Monte Carlo simulation. Portfolio returns as well as several risk-adjusted performance measures popular among financial practitioners are used for evaluation purposes. On the one hand, we find that the differences in portfolio returns between MVO and MSO are correlated with the skewness of asset returns. On the other hand, MSO consistently outperforms MVO in terms of all but one risk-adjusted performance measures as long as asset skewness is not close to zero.

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1 Introduction

For the past 40 years, Markowitz mean-variance portfolio optimization (MVO), along with the Capital Asset Pricing Model (CAPM) which is built upon it, has been some of the most widely discussed topics among both academics and practitioners in the financial industry. Markowitz optimization has won applause especially because of its simplicity: it is only concerned with two parameters, viz. the expected return and the risk. Whereas the expected return is easy to define, the measure of risk has constantly drawn debates.

Variance does not distinguish downside risks from upside risks, and a risk model based on variance will view unexpected losses and gains as similarly undesirable. The limitation discussed above can only be avoided when at least one of the following two important assumptions holds: (1) returns on financial assets follow symmetric distributions (2) investors have quadratic utility functions (Estrada. 2007). More specifically, if asset returns are symmetric, semivariance will contain the same information as variance (more on this later), and if investors' utility functions are quadratic upside and downside risks will then be similarly undesirable. However, researches in both financial econometrics and behavioral finance cast serious doubt on both of them (Bertsimas et al, 2004). Asset returns can display significant positive or negative skewness at different time frames. and most investors (excluding those with index tracking objectives) will certainly view downside risks more undesirable than upside risks.

Markowitz (1952) first proposes using semivariance as an alternative measure to variance, and he defines it as $E(min[0, R_p - T]^2)$ where Rp is the portfolio return and T is the return of a fixed benchmark. Hogan and Warren (1974) further develop Markowitz's suggestion and lay out the theoretical foundation for empirical researches. Porter (1974) shows that semivariance models are far more consistent with the stochastic dominance rules. Empirical research also lends support to the plausibility of differentiating upside risk and downside risk. Ang, Chen, and Xing (2006) find that stocks with higher downside risks bear premiums that cannot be explained by currently available symmetric pricing models. Estrada (2007) concludes that in emerging markets the CAPM based on downside beta has superior explanatory powers than traditional symmetric beta, and the paper further observes that the differences in performance are more pronounced in emerging markets than in developed markets due to the former's larger skewness.

Based on the above theoretical and empirical advantages of using semivariance over variance in various models, it is plausible to incorporate semivariance into portfolio optimization. Specifically, we can perform mean-semivariance optimization (MSO) instead of MVO by replacing variance, standard deviation, and covariance matrix with semivariance, semideviation, and semicovariance matrix (Estrada 2007). Moreover, Estrada (2007) proposes a heuristic approach for calculating semicovariance matrix that "makes mean-semivariance optimization just as easy to implement as mean-variance optimization". Computational effort should no longer hinder the practicability of MSO.

Since Estrada's "heuristic approach" levels the playing field of MVO and MSO in terms of computational power, the relative performance of MVO vs. MSO portfolios increasingly becomes the deciding factor between the two methods. It is especially worth recapping and further elaborating here that the limitation of variance as a risk measurement can potentially be avoided if distributions of asset returns are symmetric. As Estrada (2005) points out: If all distributions were symmetric, then the semideviation and the standard deviation would contain the same information. In another word, MVO and MSO will generate identical portfolios if return distributions are perfectly symmetric. And this is not unique to MSO: optimization based on shortfall, another downside risk measurement, also shares the above property (Bertsimas et al, 2004). MSO is at least as good as MVO under symmetric distributions, but its potential advantage depends on return distributions being asymmetric (Estrada 2005). Hence, it is natural for this paper to focus on skewness, which is the most common measurement of asymmetry. By varying the degree of skewness of return distributions, we not only examine if such an advantage exists but also how the level of asymmetry quantitatively impacts the scale of that advantage.

However, measuring the level of the above stated advantage is not as straight forward as it seems. In order to gauge the relative performance of MVO and MSO, we need to calculate the risk-adjusted returns for their optimal portfolios. We cannot directly compare MVO's variance-based Sharpe ratio (V-Sharpe) and MSO's semivariance-based Sharpe ratio (S-Sharpe) for the obvious reason that they use different risk measurements. It is also not proper to apply either V-Sharpe or S-Sharpe alone indistinguishably to both MVO and MSO portfolios: since MVO optimizes V-Sharpe while MSO optimizes S-Sharpe, MVO portfolio's V-Sharpe will invariably be higher than MSO portfolio's V-Sharpe given the same set of investable assets and constraints, and the reverse is true if we compare MVO and MSO's S-Sharpe.

To overcome this difficulty, instead of using one or two measurements that might potentially bias the results, we use a plethora of performance measures common in financial practices: Omega, the Sortino ratio, Kappa 3, the upside potential ratio, the Calmar ratio, the Sterling ratio, the Burke ratio, and the Modified Sharpe (as detailed in Eling and Schuhmacher, 2007).

Our main contribution to the current literature on mean-semivariance optimization is to quantitatively investigate the relationship between the skewness of asset returns and the relative performance of MVO and MSO using Monte Carlo simulation. Specifically, we present evidence showing that differences between MVO and MSO portfolio returns are statistically significant and are correlated with the skewness of asset returns. When assets are negatively skewed, MSO portfolios in average have higher returns than MVO portfolios, and the reverse is true when asset returns are positively skewed. Furthermore, if we compare MSO portfolios and MVO portfolios based on risk-adjusted performance measures, MSO portfolios invariably outperforms MVO in terms of all but one measure as long as asset skewness is not close to zero. If we combine the above two results together, we show that MSO outperforms MVO in terms of both portfolio return per se as well as risk-adjusted performance measures when asset returns are negatively skewed. On the other hand, when asset returns are positively skewed, MSO portfolios have lower returns but higher risk-adjusted performance measures.

The rest of this paper is organized as follows. In Section II, we provide a detailed description of the optimization method used for both MVO and MSO and a more detailed exposition of performance measures we use. In Section III, we use Monte Carlo simulation to systematically compare the performance of MSO and MVO under different asset return skewness levels. Section IV presents results from a series of robustness tests. Section IV concludes.

2 Methodology

2.1 Optimization Problem

A generic Markowitz style optimization problem can be expressed as the following[1]:

$$Max_{\omega_1,\omega_2,...,\omega_n} \frac{E[R_p] - R_f}{\sigma_p} = \frac{\sum_{i=1}^{n} \omega_i E[R_i] - R_f}{\sqrt[2]{\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij}}} \quad (1)$$

where $E[R_p]$ is the expected portfolio return,$R_f$is the risk free rate of return. $E[R_i]$ is the expected return of asset i, σ_p and σ_{ij} denote portfolio standard deviation and the covariance between assets i and j respectively. ω_i is the weight assigned to asset i.

The above MVO problem can be easily modified to a MSO problem by replacing standard deviation and covariance with semideviation and semicovariance. All statistics related to semivariance model are well defined in Hogan and Warren (1974) and refined in Estrada (2007). In this paper, it is worth noting that we define semivariance according to Estrada (2007)'s definition and set our benchmark to the arithmetic mean of return series, so the semivariance of portfolio returns becomes[2]:

$$\Sigma_p^2 = (1/T) \sum_{t=1}^{T} [Min(R_{pt} - \mu, 0)]^2. \quad (2)$$

and semicovariance is:

$$\Sigma_{ij} = (1/T) \sum_{t=1}^{T} [Min(R_{it} - \mu_i, 0) Min(R_{jt} - \mu_j, 0)] \quad (3)$$

For our purpose, both of our MVO and MSO follow an in-sample Markowitz direct optimization where parameter estimators of risk and return are sample statistics of the entire investment period of length T. Portfolios are not

[1]This is only one of the four standard portfolio optimization problems according to Estrada (2007). The others include:1)minimizing portfolio risk;2)minimizing portfolio risk subject to target return; and 3)maximizing return subject to target risk level. Our focus is on the problem expressed here: maximizing risk-adjusted returns, as per Estrada (2007).

[2]In terms of notation, we follow Estrada(2007): lower case σ^2 denotes variance and upper case Σ^2 denotes semivariance. All parameters expressed are in scaler form applicable to individual assets.

rebalanced, and returns are calculated using one single set of weights for all periods within T. Expected return, or realized return in our in-sample optimization, of a given portfolio is calculated as the arithmetic mean of returns across the T periods, following the precedence in Estrada (2007). We also apply the following constraints to both MVO and MSO:

$$\sum_{i=1}^{n} \omega_i = 1, \tag{4}$$

$$-2 \leqslant \omega_i \leqslant 2 \; \forall i, \tag{5}$$

where n is the number of assets in the portfolio and ω_i is the weight assigned to asset i. Equation (2) guarantees that the portfolio is fully invested, and equation (3) allows for short selling and leverage by bounding the weight of individual assets between -2 and 2.

We set our risk-free rate of return at 0 for both optimization methods. So for MVO, the optimization problem is to maximize $\frac{R_p}{\sigma_p}$, where σ_p denotes the standard deviation of portfolio returns, subject to constraints in equation (2) (3). For MSO we maximize $\frac{R_p}{\Sigma_p}$ subject to the same constraints.

2.2 Performance Measure

Our choice of performance measures follows the precedence in Eling and Schuhmacher (2007) where several approaches are listed to measure the performance of hedge funds. Those included are: Sharpe ratio, Omega, Sortino ratio, Kappa 3, upside potential ratio, Calmar ratio, Sterling ratio, Burke ratio, excess return on value at risk, Conditional Sharpe ratio, and Modified Sharpe ratio. For our purpose, we exclude Sharpe ratio because it will inevitably bias the results as discussed in previous sections. We also exclude excess return on value at risk and Conditional Sharpe ratio. They both rely on the assumption that return distributions are symmetric; however, our preliminary analysis show that both MVO and MSO portfolio returns are skewed. Instead, we include Modified Sharpe ratio, a more robust VaR based performance measure that more accurately calculates value at risk under asymmetric return distributions (Favre and Galeano, 2002). Eling and Schuhmacher (2007) categorize the above measures into three groups according to the risk measures they use: lower partial moments, drawdown, and value at risk. We will introduce them in the same order.

2.2.1 Measures based on Lower Partial Moments

Lower partial moment (LPM) calculates the negative deviations of returns R_t from a benchmark τ, and it takes different orders based on an investor's level of risk averse. LPM of order n is expressed as:

$$LPM_n(\tau) = \frac{1}{T}\sum_{t=1}^{T} max(\tau - R_t, 0)^n \tag{6}$$

The order of LPMs should be chosen based on investors' risk aversion level: the more risk averse the higher order should be used. It is also worth noting that from the expression above, LPM of order 2 is essentially semivariance at benchmark of 0. However, since we set our benchmark for calculating semivariance as the arithmetic mean of return series, they are not identical measures. Again, we set risk free rate of return at 0. So Omega, Sortino ratio, and Kappa 3, which uses LPMs of order 1, 2, or 3 are:

$$Omega_i = \frac{\mu - \tau}{LPM_1(\tau)} + 1 \tag{7}$$

$$Sortino\ ratio = \frac{\mu - \tau}{\sqrt[2]{LPM_2(\tau)}} \tag{8}$$

$$Kappa\ 3 = \frac{\mu - \tau}{\sqrt[3]{LPM_3(\tau)}} \tag{9}$$

Eling and Schuhmacher (2007) also includes upside potential ratio, which is based on the ratio of higher partial moments (HPM) of order 1 to LPM of order 2. The expression for HPM is simply the positive deviation from a benchmark return τ analogous to LPMs:

$$HPM_n(\tau) = \frac{1}{T}\sum_{t=1}^{T} max(R_t - \tau, 0)^n \tag{10}$$

So that upside potential ratio is calculated as:

$$Upside\ potential\ ratio = \frac{HPM_1(\tau)}{\sqrt[2]{LPM_2(\tau)}} \tag{11}$$

2.2.2 Measures based on drawdowns

According to Eling and Schuhmacher (2007), drawdown-based measures are popular among practitioners because they measure the ability of "continually accumulating gains while consistently limiting losses". MD of degree i, MD_i denotes the ith lowest return. And Calmer, Sterling, and Burke ratio are based on MD of different degrees:

$$Calmer\ ratio = \frac{\mu - R_f}{-MD_1} \tag{12}$$

$$Sterling\ ratio = \frac{\mu - R_f}{(1/N)\sum_{i=1}^{N} -MD_i} \tag{13}$$

$$Burke\ ratio = \frac{\mu - R_f}{\sqrt[2]{\sum_{i=1}^{N} -MD_i^2}} \tag{14}$$

2.2.3 Measure based on value at risk

We also use Modified Sharpe ratio, as introduced by Favre and Galeano (2002) and included in Eling and Schuhmacher (2007) :

$$Modified\ Sharpe = \frac{R_p}{MVaR} \tag{15}$$

where we assume 0 risk free rate of return, and

$$MVaR = W[\mu - (Z_c + \frac{1}{6}(Z_c^2 - 1)S + \frac{1}{24}(Z_c^3 - 3Z_c)K - \frac{1}{36}(2Z_c^3 - 5Z_c)S^2)\sigma] \tag{16}$$

W denotes total wealth invested, Z_c denotes the critical value for probability $(1-\alpha)$, S denotes skewness, and K excess kurtosis. The above expression is a Cornish-Fisher expansion that calculates value at risk at significance level α by taking into consideration the skewness and kurtosis of return distribution.

While both traditional VaR and MVaR are measurements designed to capture downside risks, MVaR is more robust: MVaR converges to traditional VaR when return distribution is normal and more accurately computes Value at Risk for asymmetric and fat-tailed distributions. Thus, MVaR provides exactly what a risk averse investor cares about: accurate downside risk estimate. Since neither of MVO or MSO are optimized for MVaR, it will not bias risk-adjusted return comparisons in the way variance or semivariance does by construct.

2.3 Simulation Design

To account for a number of stylized facts of the financial time series such as volatility clustering, leptokurtosis, and leverage effect, we use GJR-GARCH (1, 1) model (Bollerslev,2008). Furthermore, instead of the traditional Gaussian density our model utilizes a skewed Student's t density, as introduced by Hansen (1994), to model innovation distributions. Our choice of that asymmetric innovation density allows us to better model the high level of skewness often observed in empirical financial data (Lambert and Laurent, 2001). Model specifications are detailed below:

$$R_{i,t} = E(R_{i,t}|\Psi_{i,t-1}) + \epsilon_{i,t}, \tag{17}$$

$$\epsilon_{i,t} = \sigma_{i,t} z_{i,t}, \;\; z_{i,t} \sim \textit{skewed Student's } t\,(\nu_i, \lambda_i), \tag{18}$$

$$\sigma^2_{i,t} = \omega_i + (\alpha_i + \gamma_i I_{i,t-1})\epsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1} \tag{19}$$

$$I_{i,t-1} = \begin{cases} 0 & : \epsilon_{i,t} \geq 0 \\ 1 & : \epsilon_{i,t} < 0 \end{cases} \tag{20}$$

where $R_{i,t}$ is the return of the asset i at time t, $\Psi_{i,t-1}$ is the set of all information at time $t-1$, $\sigma^2_{i,t}$ is the conditional variance term following a GJR-GARCH (1,1) process, $z_{i,t}$ is innovation term which follows a skewed Student's t distribution specified by parameters ν_i and λ_i, and $I_{i,t-1}$ is an indicator of the sign of innovation.

We calibrate the model based on empirical observations of monthly returns of 47 country indices available in the MSCI database. The time period chosen is from Jun/2005 to May/2015, a total of 120 months. That period is both the latest data available at the time of this paper and also the most comprehensive since data for a few of developed markets prior to the starting month Jun/2005 are not included in the database.

We choose the lags for both the ARCH term and the GARCH term in the model to be 1 for parsimony and generality. For future research on the application of MSO in specific markets, model selection criterion such as Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC) should be used to determine the length of lag. All other parameters, except the number of lags, are estimated using MFE Toolbox © Kevin Sheppard, which performs model optimization by maximizing log likelihood.

2.4 Simulation Procedures

In order to investigate the effect of skewness on MSO performance while controlling for variations in other statistical properties, we need the ability to vary the skewness of asset returns and at the same time keep return means and return standard deviations unchanged. Due to the inherent nature of randomness in our simulation process, we will not be able to guarantee the level of skewness as well as means and standard deviations simply by changing or fixing certain parameters in our models. Instead, we control for those properties through a selection process using one single set of parameters: we generate more assets than we need, and then pick from them the ones that display our specified level of skewness, mean, and standard deviation.

Specifically, we first simulate the largest amount of assets based on our empirical data, which is 47 time series that represent the whole asset universe analogous to the 47 country indices within the MSCI database. Then we filter out the assets whose mean and standard deviation deviate too far from the level we set. Next, among the remaining assets we further pick those that display the level of skewness that we wish to investigate while filtering out all others. After this selection process, the remaining time series will have the specified level of skewness, mean, and standard deviation. If the number of those time series exceeds the number of assets we wish to invest in, we will randomly select from them the amount we need. On the other hand, if there are not enough remaining, we will run the entire simulation again until enough number of them are obtained.

3 Results

We divide skewness from -1 to 1 into 5 different ranges: [-1, -0.5] for high level of negative skewness, [-0.5, 0] for moderate level of negative skewness, [-0.25, 0.25] for near zero skewness, as well as [0, 0.5] and [0.5, 1] for different levels of positive skewness. We keep our mean monthly returns between [-1, 1][3] and standard deviation between [6, 8]. We calibrate the ranges for skewness based on both empirical observations and theoretical support, and we control the range for mean and standard deviations as narrow as possible

[3] All returns in this paper are in percentages.

without remarkably decreasing computational efficiency. [4]

For each skewness level, we obtain 5 assets per simulation run according to the filtering technique described in the last section, and each asset has a return series of 1000 periods. Then we generate one MSO and one MVO portfolio from the simulated assets. We run this simulation and optimization process for 1000 times. So in total we have 1000 MSO portfolios and 1000 MVO portfolios for each skewness level. All results corresponding to one specific skewness level are then the average value of such 1000 portfolios.

For all of our performance measures, we assume benchmark τ to be equal to the risk free rate which then subsequently equals to 0. For drawdowns in Sterling ratio and Burke ratio, we consider drawdowns till the 5th largest. or $N = 5$. For Modified Sharpe ratio,we evaluate value at risk at 0.05 significance level, thus $\alpha = 0.05$ and $Z_c = -1.96$ (following the convention in Eling and Schuhmacher, 2007).We also set our initial wealth $W = 1$.

Furthermore, we also test for the statistical significance of the differences between MSO and MVO by performing the sign test, a non-parametric statistical hypothesis test that compares two matched samples. It tests the null hypothesis that the median of sample differentials are zero, i.e. the median of the differences between two samples are zero. It can also be interpreted as testing if in each pair the two observations have equal chance of being larger than the other (Diebold and Mariano, 2002) In our context, a rejection will denote that one portfolio is more likely to outperform the other. It does not rely on any assumption on population distribution so it is often used as an alternative to the paired Student's t-test for paired samples from unknown distributions. It fits our sample because each pair of our portfolios, 1000 of them in total, are matched in the sense that one pair of MSO and MVO portfolios are optimized based on the same asset return series. We acknowledge that this test is not specifically designed for testing the risk-adjusted performance measures we use, though our sample meets all its assumptions.[5]

[4]The 90% range for our assets' skewness, mean, and standard deviation are [-0.12,0.85],[-0.21,1.34],and [4.81,10.85] respectively. Though there are few positive skewness series within our empirical data,on longer scales returns do tend to skew to the right(Favre and Galeano, 2002). We allowed for more negative variations in the mean due to the randomness of our GARCH process, and we are able to control for standard deviation into a quite narrow range within our empirical range.

[5]Other assumptions besides that data are paired include: 1)each pair is chosen randomly and independently: and 2)data are measured on an ordinal or continuous scale instead of nominal. Each pair of portfolios are indeed independent of other pairs since each pair are optimized for different repetitions of simulated asset return series and each

observe in Table 1 are the average of a very consistent series of small scale individual observations.

Furthermore, it is also worth noting that the count of larger-than-zero observations does not change much as skewness moves from more extreme levels to moderate levels. Around 75% of MSO portfolios outperform MVO portfolios when skewness are negative in both [-1,-0.5] range and [-0.5,0] range, and around the same proportion of underperformance under both positive skewness ranges. Combined with observations from Table 1, the level of skewness only affects the scale of the differences but have no effect on the probability of observing an MSO outperformance or underperformance as long as skewness is not close to zero, in which case we see that the chance is around 50%.

[Figure 2]

[Figure 3]

Figure 2 further presents MSO and MVO portfolios' cumulative return over the entire investment length, and Figure 3 is a zoom-in for performance in the first 120 periods.

4 Robustness Test

4.1 Correlation check

As a robustness check, we control for cross correlations among assets. In our univariate GJR-GARCH model, cross correlations between assets are practically zero, all lying between -0.1 and 0.1. However, it is also not uncommon for financial time series to exhibit far larger cross correlations during different time frames or in different asset classes. And we recognize that cross correlation might have an impact on our results, so we use a multivariate Constant Conditional Correlation (CCC) -GJR-GARCH (1,1) with skewed Student's t innovations to model assets that display significant cross correlations (Bollerslev, 2008; He and Tersvirta, 2004). The model is specified as below:

$$R_t = E(R_t|I_{t-1}) + \epsilon_t, \tag{21}$$

$$\epsilon_t = H_t^{1/2} z_t, \; z_t \sim \; skewed \; Student's \; t \; (\nu, \lambda), \tag{22}$$

$$H_t = D_t P D_t, \tag{23}$$

$$P = [\rho_{ij}], \tag{24}$$

$$D_t = diag(h_{1,t}^{1/2}, ..., h_{N,t}^{1/2}), \tag{25}$$

$$h_{i,t} = \omega_i + (\alpha_i + \gamma_i I_{i,t-1})\epsilon_{i,t-1}^2 + \beta h_{i,t-1}, \tag{26}$$

Rt is a N by 1 vector for returns on all assets at time t, ϵ_t a N by 1 vector containing the error terms for all assets, H_t is the conditional variance-covariance matrix whose entries are generated through a GJR-GARCH (1, 1) process, and P is the constant conditional correlation matrix.

The same data are used for calibrating parameters in this model as in the univariate model. However, estimation methods are a bit more nuanced. Due to a lack of log likelihood function within the MFE Toolbox for multivariate models coupled with skewed Student's t distribution, parameters except for ν and λ are first estimated through maximizing the log likelihood of CCC-GJR-GARCH (1, 1) with Gaussian innovation, and then we use the same parameter specifications for ν and λ obtained through the previous univariate model to guard our skewed Student's t innovation process for each asset in this multivariate model. This not only guarantees that our multivariate model still captures the stylized facts about financial time series as discussed in previous sections, but also enables us to control for skewness while modeling high levels of cross correlation. The average correlation matrix across all repetitions for 5 assets in our model is:

$$\begin{bmatrix} 1 & 0.69 & 0.69 & 0.68 & 0.66 \\ 0.69 & 1 & 0.68 & 0.67 & 0.66 \\ 0.69 & 0.68 & 1 & 0.68 & 0.67 \\ 0.68 & 0.67 & 0.68 & 1 & 0.65 \\ 0.66 & 0.66 & 0.67 & 0.65 & 1 \end{bmatrix}$$

We also follow the same simulation procedures as in our univariate model, only changing the total number of repetitions from 1000 to 200 for computational efficiency. We still simulate a universe a 47 assets with 1000 periods of returns and then use the same selection process to pick the five that we will invest in, and we run it for 1000 times. Results reported below are the average of those 1000 repetitions.

Our results show that using the CCC model, MSO and MVO do not display any practically significant differences in terms of all performance measures including portfolio returns. All differences in returns are less than 1 basis point. This result is not entirely unexpected. Since the assets in CCC model all display a correlation around 0.7 as per our calculation, different assets will exhibit similar levels of downside risks as well as similar upside risks. Thus, the portfolio cannot be well diversified with either MVO or MSO, and their returns will hence be much more aligned with each other.

4.2 Mean and standard deviation check

In this section we expand the range of mean and standard deviation to intentionally introduce more noise into our asset return series, and we investigate the relative performance between MSO and MVO with the same methodology in previous sections. The results are shown in Exhibit 2.

[Exhibit 2]

From Exhibit 2 we can observe that all the entries remain qualitatively the same as in Exhibit 1: no signs have changed. Quantitatively, however, there is a difference between the effects from mean and standard deviation. In Panel A, we expand the range for mean from our previous [-1,1] to [-2,2] and [-3,3] respectively while keeping the range of standard deviation still within [6,8]. Numerically speaking, differences in all performance measures increase significantly. It is also worth noting that the z-scores reported from sign test are considerably lower, though still significant at 0.01. However, it does not necessarily indicate that the relative performances between MSO and MVO are less consistent. More likely it is the result of decreasing sample size in our robustness test. Indeed, per our calculation the proportions of MSO outperformances in portfolio return at skewness of [-1,-0.5] are 82% and 76% for mean at [-2,2] and [-3,3] respectively. They are in line with the proportions we observe in the previous section, if not higher. As for broader ranges of standard deviations, we do not find much change in performance compared with observations in Exhibit 1.

5 Conclusion

Ever since the adoption of Markowitz-style mean-variance optimization for asset allocation, the method has been under criticism for its lack of distinction between downside risks and upside risks. Markowitz first proposes using semivariance to substitute for variance, and literature has so far built on his proposal and laid the groundwork for putting semivariance into practice. This paper contributes to the current literature by systematically investigating how mean-semivariance optimization behaves compared with conventional mean-variance optimization under different asset distribution settings. To the center of our research, we examine the effect of asset return skewness since various available literature points to potential benefits of using semivariance models in skewed markets.

Our results show that MSO outperforms MVO in terms of all but one performance measures as long as asset skewness is not close to zero. We also observe a consistent negative relationship between asset skewness and the difference between MVO and MSO portfolio returns: the lower the skewness, the larger the scale of MSO outperformance. However, the chance for such outperformance to occur does not change with skewness. Furthermore, our robustness test results show that high levels of cross correlation among assets will practically eliminate the differences between MSO and MVO in terms of all performance measures. Nonetheless, MSO is at least as good as MVO. The introduction of more noise in mean, on the other hand, dramatically increases the differences between MSO and MVO without sacrificing the level of consistency.

In practice, mean-semivariance optimization is still not widely used partly due to a lack of understanding of the relative performance between MSO and the more familiar MVO. This paper presents results that a risk-averse investor whose utility aligns more closely with alternative performance measures than traditional Sharpe ratio should definitely apply MSO rather than MVO, further considering that MSO and MVO now require nearly identical levels of computational power. Moreover, for practitioners who are either risk neutral or whose utility entails more considerations than the measurements used in this paper, the distribution of return differences between MSO and MVO will certainly be beneficial in making more informed investment decisions.

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Figure 1: Distribution of 1000 MVO / MSO Potfolio Return Differences

The five figures below show the histogram for the difference between 1000 pairs of MSO and MVO portfolio returns. Figures differ by asset skewness ranges. Differences are calculated as MSO return minus MVO return. '>0 count' donates the number, out of 1000, that the return difference is larger than zero. 's' denotes skewness level.

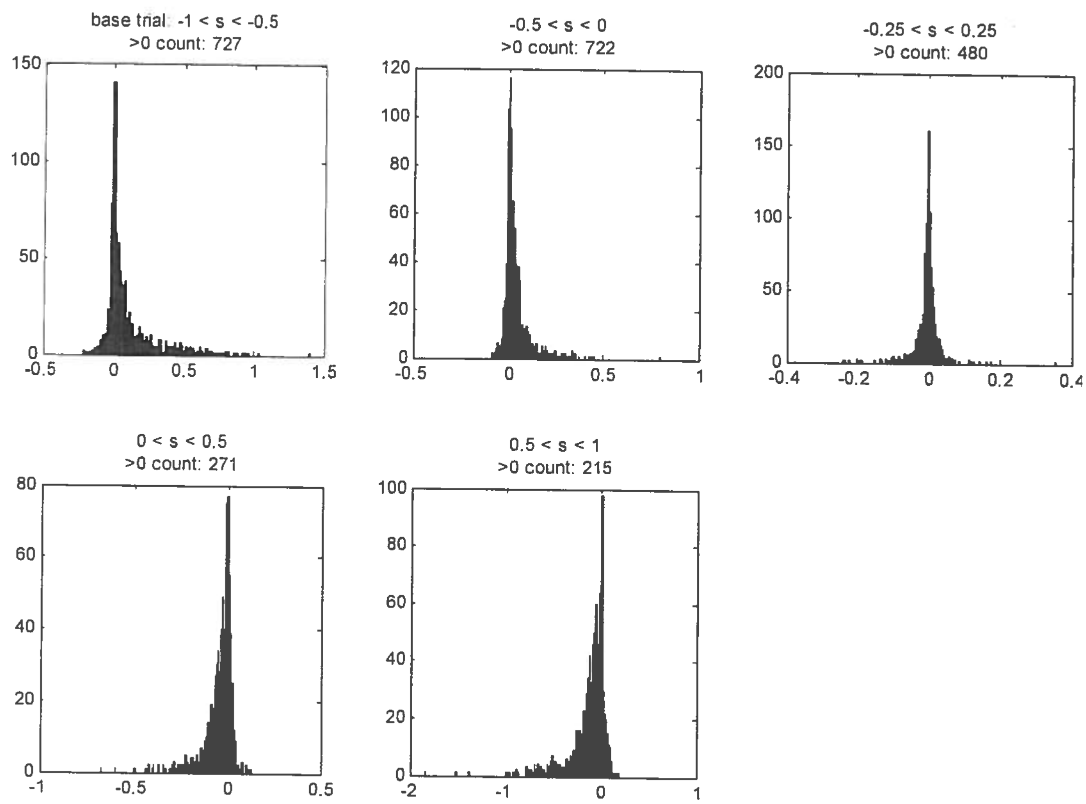

Figure 2: Cumulative Performance of 1000 MVO/MSO Potfolio Returns

The five figures below show the cumulative MVO and MSO portfolio performance through 1000 periods. The start point is 1 dollar for both portfolios. Figures differ by level of skewness, denoted as *s*. Since we repeated 1000 times for each skewness level, return for each single period is the average value of 1000 simulation runs.

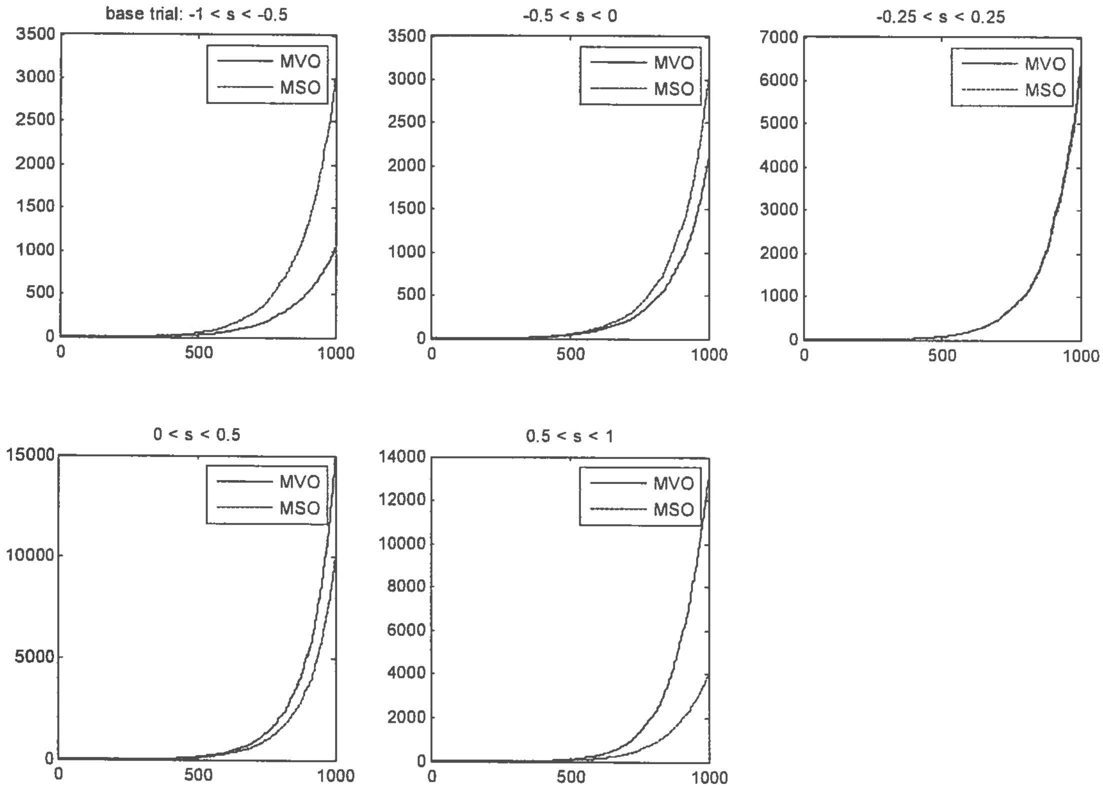

Figure 3: Cumulative Performance of 120 MVO/MSO Potfolio Returns

The five figures below show the cumulative MVO and MSO portfolio performance among the first 120 months. The start point is 1 dollar for both optimazition methods. Figures differ by asset skewness ranges, denoted as s. Since we simulated 1000 times for each skewness level, performance for each period is the average value of 1000 simulation runs.