Toward an Efficiency Rationale for the Public Provision of Private Goods*

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Abstract

We show that public provision of private goods may be justified on pure efficiency grounds. A government's involvement in the provision of the private good generates information about individuals' private good purchases that facilitates more efficient revenue extraction for the provision of public goods. Public provision of the private good improves economic efficiency under a condition that is always fulfilled under independence and satisfied for an open set of joint distributions. The efficiency gains from public provision of private goods require that consumers cannot arbitrage, so our analysis applies to private goods where it is easy to keep track of the ultimate user, such as schooling and health care, but not to easily tradable consumer goods.

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1 Introduction

Governments at all levels not only provide public goods, but also devote considerable resources to the provision of private goods such as health care, housing and education. This is puzzling in the light of the standard public finance justifications for government intervention in the marketplace. Over the last twenty years a number of models have been developed to rationalize public provision of private goods. All such models ultimately explain it as an *instrument to redistribute* from the wealthy to the poor.[1]

The contribution of this paper is to demonstrate that public provision of a private good can also be welfare improving in an environment in which there are no preferences for redistribution. Since several publicly-provided private goods, such as publicly provided higher education, have neutral or regressive distributional effects, we believe that our explanation complements the existing literature in a highly relevant dimension.

We adopt a mechanism design approach, where we compare the welfare under *some* joint provision mechanism with the *best* outcome that can be implemented under separate provisions. This approach avoids results that hinge on explicit or implicit ad hoc restrictions on how the private good market operates.

Just like some mechanism design papers in the literature that is based on redistributive preferences, our results require that consumers cannot trade the private good ex post. Such arbitrage would restore a uniform price for the private good and therefore destroy our bundling mechanism. While this property is not unique to our paper, we nevertheless consider it a desirable property, as it creates a distinction between commodity-like goods –which could be left to the market– and services where the identity of the final consumer is observable –which could be useful for incentive reasons. The no arbitrage restriction also seems realistic for many goods that are publicly provided in the real world. For example, publicly provided education (including higher education), public health insurance, and public health care, are all difficult to resell.

Our analysis also illustrates a more general point. Since Ramsey (1927), it has been standard in the optimal taxation literature to ask how to best raise a given target revenue without considering what the tax revenue is intended for. Similarly, public good provision and other public expenditure problems always take the size of any outside funds as given. This paper provides an example where the taxation and expenditure problems are non-separable: the optimal commodity tax to finance the public good should depend on whether the consumer gets access to the public good.[2] Hence, the analytically convenient dichotomy between government expenditures and revenue may result in an efficiency loss.

We adapt ideas that are well-known in the literature on commodity bundling to a welfare optimization problem involving both public and private goods, and show that public provision of

[1] An exception is a recent paper by Manski (2009) which provides an argument for collective choice based on the mathematics (Jensen's inequality) of aggregating individual payoffs in an environment of private decision making under uncertainty.

[2] Related points are made in Boadway *et al.* (1998) and Blomquist *et al* (2008).

private goods can be explained as an *instrument to improve economic efficiency.* Using publicly-provided higher education as a concrete example, our model implies that, in an efficient allocation, the in-state tuition for college education (a private good) should be below the out-of-state tuition to provide incentives for the citizens (i.e. parents of current and/or future college students) to stay in the State and pay the taxes that finance the local public goods.

We model the government as a benevolent social planner operating in an economy with both private and excludable public goods. To avoid trivialities we assume that there are no complementarities on either the production or the consumption side. As a result, the complete information social optimum is fully separable, with no elements of bundling.

Consumers are privately informed about their preferences, implying that the planner is constrained to use a mechanism that provides incentives for truth-telling. Together with participation constraints, which rule out financing the public good through lump sum taxes, the informational asymmetry makes the complete information social optimum impossible to implement.

As a benchmark, we consider the case where *by assumption* the public good and the private good are provided separately. That is, each good is allocated without use of individual data from the other market, thus ruling out mechanisms, for example, where the price charged for the private good depends on whether the individual is willing to pay the user fee for the public good, but allowing for cross subsidies. To interpret this *separate provision problem* one may imagine a setup where two independent government agencies are separately in charge of the public and private good provisions, but where the government as a whole faces an integrated budget constraint. Besides the restriction that information cannot be shared between the two agencies, we impose no other restrictions on the feasible mechanisms. However, using arguments from the optimal auction literature we provide a very simple characterization of the best separate provision mechanism, which can be described as an access fee for the public good, a provision probability of the public good, and a price for the private good. Thus the best separate provision mechanism is fully consistent with a decentralized market for the private good as all the agency in charge of the private good market needs to do is to set a linear sales tax for the private good.

We contrast the best separate provision mechanism with the case where the goods are jointly provided. By "joint provision," we mean that *all information* can be used for decisions on pricing and provision of *both* goods. This leads to a model where types are multidimensional, thus preventing us from characterizing the constrained optimal mechanism due to well-understood technical difficulties. Fortunately, our primary interest is the qualitative question of *when public provision of the private good is welfare improving.* For this, it is sufficient to demonstrate that there is *some joint provision mechanism that outperforms the best separate provision mechanism.* We thus consider small deviations from the best separate provision mechanism and furthermore restrict ourselves to deviations characterized by three fixed prices, one for the private good only, one for the public good only, and one for the bundle consisting of both goods. Under a condition, which is satisfied by a large set of joint distributions, including the case where valuations are independent, there indeed exist deviations that improve upon the best separate provision mechanism. The welfare improving mechanism we identify is generally not constrained optimal because we are making a local argument using only a particular class of pricing mechanisms, but the constrained optimal mechanism must weakly outperform the simple mechanism under our consideration. Hence the optimal joint provision mechanism *must condition prices for private good consumption on valuations for the public good*. This *cannot be implemented using an anonymous market mechanism for the private good*, which is why we interpret non-separability between the markets as public provision.

The remainder of the paper is structured as follows. Section 2 provides a motivating example. The best separate provision mechanism is characterized in Section 3. The main result, which establishes that there is public provision of private goods in the optimal mechanism, is presented in Section 4. Section 5 reviews the related literature in greater details and explains our contribution relative existing models and Section 6 concludes.

2 An Example

Consider an environment with a unit-mass continuum of ex ante identical consumers. Consumers have preferences over two goods, an excludable public good and a private good, and utility is transferable. We let θ_G denote the willingness to pay for the public good and θ_P denote the willingness to pay for the private good, and assume for now that θ_G and θ_P are independent random variables distributed uniformly on the unit interval $[0, 1]$.

The distinction between the public good and the private good is that the public good can be provided to everyone in the economy by incurring a fixed cost K, whereas the private good is produced at constant unit cost c.

As a first step, imagine that the public good must be self-financing. Let π be the probability that the good is provided and f be the user fee.[3] The best self-financing mechanism $\langle \pi, f \rangle$ is thus the solution to the problem

$$\max_{\{\pi, f\}} \pi \int_f^1 \theta_G d\theta_G \tag{1}$$

$$\text{s.t.} \qquad f(1-f) \geq \pi K, \tag{2}$$

which has solution

$$\pi^* = \begin{cases} 1 & \text{if } K \leq \frac{1}{4} \\ 0 & \text{if } K > \frac{1}{4}, \end{cases} \qquad f^* = \begin{cases} \frac{1}{2} - \sqrt{\frac{1}{4} - K} & \text{if } K \leq \frac{1}{4} \\ 0 & \text{if } K > \frac{1}{4}. \end{cases} \tag{3}$$

To understand (3), note that a profit maximizer would set access fee $f = \frac{1}{2}$, resulting in maximized revenue of $\frac{1}{4}$. Hence, it is possible to provide the good and break even if and only if $K \leq \frac{1}{4}$, and, from a social standpoint, it is always desirable to provide the public good if K is in this range. The constrained optimal fee is then obtained by solving the quadratic equation one obtains by setting $\pi = 1$ and requiring that constraint (2) is satisfied with equality.

[3] In our general analysis we allow for much more general mechanisms, but it turns out that linear pricing is optimal when goods are provided separately (see Proposition 1 below).

Next, suppose that, in the spirit of Ramsey taxation, a government uses revenue from a sales tax on the private good to subsidize access to the public good. Assuming that T dollars are collected from the private good sales tax and that randomizations are not allowed, the public good provision problem is just like (1) with fixed cost $K-T$.[4] Hence, the best pricing and provision scheme is

$$\pi(T)=\begin{cases}1 & \text{if } K\leq \frac{1}{4}+T\\ 0 & \text{if } K>\frac{1}{4}+T,\end{cases}\qquad f(T)=\begin{cases}\frac{1}{2}-\sqrt{\frac{1}{4}+T-K} & \text{if } K\leq \frac{1}{4}+T\\ 0 & \text{if } K>\frac{1}{4}+T.\end{cases} \tag{4}$$

In order to raise T we have to set a unit sales tax t on the private good satisfying $T=t(1-c-t)$. Solving the quadratic for its smallest root we find that the relationship between the tax revenue and the associated tax rate is

$$t=\frac{1-c}{2}-\sqrt{\left(\frac{1-c}{2}\right)^2-T}. \tag{5}$$

This is a valid solution for our problem if and only if T is less than $\left(\frac{1-c}{2}\right)^2$, which is what a profit maximizing monopolist would earn if selling the private good.

In order to set the sales tax on the private good optimally, the benevolent planner must trade off the distortion created in the market for the private good with the efficiency gains from being able to lower the user fees for access to the public good. This optimization problem can be formulated as

$$\max_{0\leq T\leq\left(\frac{1-c}{2}\right)^2}\int_{\frac{1}{2}-\sqrt{\frac{1}{4}+T-K}}^{1}\theta_G d\theta_G+\int_{c+\left[\frac{1-c}{2}-\sqrt{\left(\frac{1-c}{2}\right)^2-T}\right]}^{1}(\theta_P-c)\,d\theta_P. \tag{6}$$

It is easy to verify from the first order condition that the unique solution to (6) is to set the subsidy to the public good as

$$T^*=\frac{(1-c)^2K}{1+(1-c)^2}. \tag{7}$$

Note that T^* –the share of the cost for the public good that is financed by taxing the private good – is decreasing in the unit cost of the private good, which simply reflects that fewer consumers will purchase the good when the price is higher, so it becomes harder to generate tax revenue.

From now on we will specialize the example further to simplify the algebra by assuming that $c=0$, so that $T^*=\frac{K}{2}$. The maximized welfare in (6) can then be directly calculated as

$$\frac{1}{2}\left[\theta_G^2\right]\Big|_{\frac{1}{2}-\sqrt{\frac{1}{4}+T^*-K}}^{1}+\frac{1}{2}\left[\theta_P^2\right]\Big|_{\frac{1}{2}-\sqrt{\frac{1}{4}-T^*}}^{1}-K=\frac{1}{2}\left(1+\sqrt{1-2K}+K\right)-K. \tag{8}$$

While T^* corresponds to an optimal tax, we will now demonstrate that a regime that provides the private good for free to all tax-paying citizens can do better in terms of welfare.[5] Here we will consider a "pure bundling mechanism", and it will only dominate the regime with Ramsey taxation

[4] When $K>\frac{1}{4}+T$ randomizations will be part of an optimal mechanism.

[5] The reader may worry that this is because we only consider a linear tax. However, our general analysis establishes that a linear tax is without loss of generality in the "Ramsey tax regime" (again, see Proposition 1 below).

in particular parameter regions. However, our general analysis will show that an element of public provision of the private good is generically desirable.

If the only way to get access to the private good is to opt in to the government bundle, the relevant willingness to pay is that for joint consumption of the two goods. This joint willingness to pay, denoted by $\theta = \theta_G + \theta_P$, is distributed on $[0, 2]$ in accordance with the symmetric triangular density function

$$h(\theta) = \begin{cases} \theta & \theta \in [0,1] \\ 2-\theta & \theta \in [1,2]. \end{cases} \tag{9}$$

Let b denote the access fee to the private-public good bundle. Because the maximal revenue under the density (9) is attained at $b < 1$ we only consider K that are small enough so that the best bundle price solves the equation

$$[1 - H(b)]\, b = \left(1 - \frac{b^2}{2}\right) b = K. \tag{10}$$

For concreteness, let $K = 7/16$, which is convenient because it translates into an optimal bundle price of $1/2$. This implies that only $1/8$ of the consumers are excluded from consumption of the bundle. Recall that for $K = 7/16$ in the separate provision taxes-and-subsidies regime, the use fee for the public good and the sales tax of the private good as given by (4) and (5) are (by setting $c = 0$, and $T = T^* = K/2 = 7/32$) :

$$t^* = f^* = \frac{1}{2} - \sqrt{\frac{1}{32}} \approx 0.323 \tag{11}$$

Hence, the prices under separate provision are higher than the price for the bundle on a per good basis. This implies that there are fewer exclusions from usage under bundling than under the benchmark tax and subsidy regime. The reason for this is that the tails of the distribution of willingness to pay are fatter when goods are provided separately than with bundling.[6] Clearly, fewer exclusions is a beneficial effect from providing the private good jointly with the public good. However, there is also a negative effect that is created by the cruder pricing scheme when goods are jointly provided. Some agents with a high valuation for the private good may decide to opt out of the bundle because they have a low valuation of the pubic good and vice versa. Obviously, this generates a misallocation that tends to make separate provision more desirable. In general, either effect can dominate, but for the example with $K = 7/16$ the welfare gain from the reduction use exclusions is large enough to make the regime with public provision of the private good more desirable.

To see this, we calculate the welfare in the regime where the government jointly provides the two goods at price $b = 1/2$ as

$$\int_{\frac{1}{2}}^{1} \theta^2 d\theta + \int_{1}^{2} \theta (2-\theta)\, d\theta - K = \frac{23}{24} - \frac{7}{16} \approx .9583 - \frac{7}{16}, \tag{12}$$

[6] See Fang and Norman (2006) for details.

which is to be compared with the welfare under the regime with taxes and subsidies (from (8 by setting $K = 7/16$),

$$\frac{1}{2}\left(1+\sqrt{1-2\frac{7}{16}}+\frac{7}{16}\right)-K=\frac{23+4\sqrt{2}}{32}-\frac{7}{16}\approx 0.89-\frac{7}{16}. \tag{13}$$

3 The General Case: Separate Provision Mechanisms

Just like in the example, an economy is populated by a continuum of *ex ante* identical agents with preferences over a binary and excludable public good and a binary private good. Utility is transferable and the public good can be produced at a per capita cost $K > 0$, whereas the private good is produced at unit cost $c > 0$.

An agent is characterized by her type $\boldsymbol{\theta} \equiv (\theta_G, \theta_P) \in \boldsymbol{\Theta} \equiv \Theta_G \times \Theta_P = [\underline{\theta}_G, \overline{\theta}_G] \times [\underline{\theta}_P, \overline{\theta}_P]$ where θ_G is her valuation for the public good and θ_P is her valuation for the private good, and these valuations are her private information. To avoid trivialities, we assume that $0 \le c < \overline{\theta}_P$ and $0 < K < \overline{\theta}_G$. We assume that the probability measure over $\boldsymbol{\Theta}$ can be represented by a smooth cumulative distribution $H : \boldsymbol{\Theta} \to [0,1]$, and write $H_G : \Theta_G \to [0,1]$ and $H_P : \Theta_P \to [0,1]$ the respective marginal cumulative distributions. Probability density functions exist and are denoted by h, h_G and h_P respectively.

Agents are risk neutral with expected payoffs given by

$$\phi_G\theta_G + \phi_P\theta_P - m, \tag{14}$$

where ϕ_G is the probability of consuming the public good, ϕ_P is the probability of consuming the private good and m is the transfer.

We begin our analysis by considering a benchmark problem where pricing and access rules must be independent across the two goods. Formally, a *separate provision mechanism* is a mechanism $(\pi, \phi_G, t_G, \phi_P, t_P)$, where $\pi \in [0,1]$ is the probability that the public good is provided; $\phi_G : \Theta_G \to [0,1]$ is the probability of access to the public good; $t_G : \Theta_G \to \mathrm{R}$ is the fee for consuming the public good; $\phi_P : \Theta_P \to [0,1]$ is the probability of consuming the private good; and $t_P : \Theta_P \to \mathrm{R}$ is the fee for consuming the private good. The reader may note that allowing π to be a non-degenerate randomization is more than a question of generality. It may very well be that the optimal mechanism involves randomization. The reason is that it may be either impossible or too distortionary for the private good market to provide the good for sure, but, by randomizing, one can get a desirable outcome at least with positive probability.

It is very important to notice that we have assumed here that (ϕ_G, t_G) are functions of θ_G only and *cannot depend on* θ_P, and that (ϕ_P, t_P) are functions of θ_P only and *cannot depend on* θ_G. This independence across goods is the defining property of a separate provision mechanism, and may be justified if markets are physically separated in space and if the government agencies cannot track the behavior of individual agents across markets. Alternatively, the constraint may simply reflect a belief that one should not intervene in the market for private goods beyond imposing a sales tax.

Note that the provision probability π is a scalar and that $(\phi_G, t_G, \phi_P, t_P)$ are all independent of the distribution of realized types. This specification is a direct result of modelling the set of agents as a continuum, which simplifies the analysis tremendously relative to the corresponding finite agent model, where all decision rules depend on the full type profile. However, it has been shown in closely related models that the continuum specification is a good approximation of the model with a large finite agent economy (see Schmitz 1997, Hellwig 2003, Norman 2004, and Fang and Norman 2010).[7]. The reason, roughly speaking, is that most agents must have a negligible influence on a *public good problem* in a large economy, thus creating a situation similar to the "Paradox of Voting."

3.1 The Problem

Given a separate provision mechanism $(\pi, \phi_G, t_G, \phi_P, t_P)$, the *ex ante* expected average utility of all the agents in the economy is:

$$\begin{aligned} & \int_{\Theta_G} \int_{\Theta_P} \left[\phi_G(\theta_G)\theta_G - t_G(\theta_G) + \phi_P(\theta_P)\theta_P - t_P(\theta_P)\right] dH(\theta_G, \theta_P) \\ = & \sum_{J=G,P} \int_{\Theta_J} \left[\phi_J(\theta_J)\theta_J - t_J(\theta_J)\right] dH_J(\theta_J). \end{aligned} \tag{15}$$

Equation (15) illustrates the fact that the welfare criterion under the separate provision mechanism is a sum of two terms that separate completely across the two markets. The best separate provision mechanism must therefore solve the following problem:

$$\max_{\{\pi,(\phi_J,t_J)_{J\in\{P,G\}}\}} \sum_{J\in\{G,P\}} \int_{\Theta_J} \left[\phi_J(\theta_J)\theta_J - t_J(\theta_J)\right] dH_J(\theta_J) \tag{16a}$$

$$\text{s.t.} \quad \sum_{J\in\{G,P\}} \left[\phi_J(\theta_J)\theta_J - t_J(\theta_J)\right] \geq \sum_{J\in\{G,P\}} \left[\phi_J(\widehat{\theta}_J)\theta_J - t_J(\widehat{\theta}_J)\right] \qquad \forall \boldsymbol{\theta}, \widehat{\boldsymbol{\theta}} \in \boldsymbol{\Theta} \tag{16b}$$

$$\sum_{J\in\{G,P\}} \left[\phi_J(\theta_J)\theta_J - t_J(\theta_J)\right] \geq 0 \qquad \forall \boldsymbol{\theta} \in \boldsymbol{\Theta} \tag{16c}$$

$$K\pi + \int_{\Theta_P} c\phi_P(\theta)\, dH_P(\theta_P) \leq \int_{\Theta_J} t_J(\theta_J)\, dH_J(\theta_J) \tag{16d}$$

$$0 \leq \phi_G(\theta_G) \leq \pi \qquad \forall \theta_G \in \Theta_G, \tag{16e}$$

where (16b) represents the incentive compatibility constraints for truth-telling, and (16c) represents the participation constraints which ensure that no agent is made strictly worse off than opting out. The latter constraint implies that the lowest type must be provided with a minimum utility level, which is what makes lump sum taxes infeasible. The resource constraint (16d) guarantees that the total costs for the production of the public and the private goods do not exceed the total revenue collected from the agents, and, finally, constraint (16e) restricts the probability of accessing the

[7]This is not always the case for mechanism design problems. For example, Groves and other pivot mechanisms cannot even be formulated when there is a continuum of agents, because it is impossible to make individual agents pivotal while at the same time maintaining measurability.

public good to be no more than the probability that the public good is provided.[8]

While the other constraints are non-controversial, the use of participation constraints in public finance is often open to some debate. Indeed, it is sometimes argued that the power of taxation implies that participation constraints are irrelevant. However, in the context of local governments it is clear that opting out can be interpreted as a shorthand for moving to a different jurisdiction. Additionally, even in the case of a federal government with no migration the participation constraint could be interpreted as a (very special) form of inequality aversion or as a minimum sustenance level. Finally, if interpreting the problem as representing the "constitutional stage", participation constraints are again appropriate.

3.2 The Characterization

Note that an important implication of the restriction to separate provision mechanisms is that the incentive compatibility constraints (16b) hold if and only if, for $J \in \{G, P\}$,

$$\phi_J(\theta_J)\theta_J - t_J(\theta_J) \geq \phi_J(\widehat{\theta}_J)\theta_J - t_J(\widehat{\theta}_J), \quad \forall \theta_J, \widehat{\theta}_J \in \Theta_J. \tag{17}$$

That is, incentive compatibility separates into "public good incentive compatibility" and "private good incentive compatibility."

More importantly, the maximization problem (16) can be decomposed into two separate unidimensional mechanism design problems. These two unidimensional problems are very similar to the optimal auction problem in Myerson (1981) and involve a term that has been known as "virtual valuation," which is defined as

$$x_J(\theta_J) \equiv \theta_J - \frac{1 - H_J(\theta_J)}{h_J(\theta_J)}. \tag{18}$$

Specifically, let $(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*)$ be a solution to problem (16); the two unidimensional problems can be defined as follows. The first is an excludable public good provision problem taking (ϕ_P^*, t_P^*) as given:

$$\max_{\{\pi, \phi_G\}} \int_{\Theta_G} [\phi_G(\theta_G)\theta_G]\, dH_G(\theta_P) - K\pi \tag{19a}$$

$$\text{s.t. } 0 \leq \int_{\Theta_G} \phi_G(\theta_G) x_G(\theta_G)\, dH_G(\theta_G) - K\pi + \int_{\Theta_P} [t_P^*(\theta_P) - c\phi_P^*(\theta_P)]\, dH_P(\theta_P) \tag{19b}$$

$$0 \leq \phi_G(\theta_G) \leq \pi \text{ for all } \theta_G \tag{19c}$$

$$\phi_G(\cdot) \text{ is weakly increasing.} \tag{19d}$$

And the second is a private good allocation problem taking (π^*, ϕ_G^*, t_G^*) as given:

$$\max_{\{\phi_P\}} \int_{\Theta_P} [\phi_P(\theta_P)(\theta_P - c)]\, dH_P(\theta_P) \tag{20a}$$

$$\text{s.t. } 0 \leq \int_{\Theta_P} \phi_P(\theta_P) x_P(\theta_P)\, dH_P(\theta_P) + \int_{\Theta_G} t_G^*(\theta_G)\, dH_G(\theta_G) - K\pi^* \tag{20b}$$

$$0 \leq \phi_P(\theta_P) \leq 1 \text{ for all } \theta_P \tag{20c}$$

$$\theta_P(\cdot) \text{ is weakly increasing.} \tag{20d}$$

[8]Note that the cost to provide the public good depends on the probability of provision π, but not on the probability of consumption $\phi_G(\cdot)$. Also note that (16d) only requires that the resources are balanced in expectation, but this is without loss of generality because one can easily adjust transfers without changing the interim expected payoffs in such a way as to balance the budget *ex post* for sure (see, e.g., Borgers and Norman 2008).

The following lemma, whose detailed derivation is in the appendix, provides the link between the original planning problem (16) and the two separate unidimensional problems (19) and (20):

Lemma 1 *If* $(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*)$ *solves the planning problem (16), then* (π^*, ϕ_G^*) *solves (19) and* ϕ_P^* *solves (20); moreover, for* $J \in \{G, B\}$, $t_J^*(\theta_J) = \theta_J \phi_J^*(\theta_J) - \int_{\underline{\theta}_J}^{\theta_J} \phi_J^*(x)\, dx$ *for all* $\theta_J \in \Theta_J$.

To facilitate further interpretations, observe that the problem for a *profit maximizing monopolist* for the public good would be to maximize

$$\int_{\Theta_G} \phi_G(\theta_G)\, x_G(\theta_G)\, dH_G(\theta_G) - K\pi \tag{21}$$

subject only to the constraints (19c) and (19d). For this problem, the "no-haggling" logic of Stokey (1979), Myerson (1981) and Riley and Zeckhauser (1983) immediately implies that the profit-maximizing mechanism is, without loss of generality, one where the monopolist charges a single price. However, this result does not extend to our problem where profits appear as a constraint in (19b). In general, the solution to the problem (19), for example, may very well be a randomized mechanism.[9] To rule out randomization in the optimum, we will assume that the "virtual valuation" $x_J(\theta_J)$ as defined in (18) is weakly increasing in θ_J, which is a standard regularity condition. Then

Lemma 2 *Suppose that* $x_J(\theta_J)$ *as defined in (18) is weakly increasing in* θ_J *for* $J \in \{G, P\}$.

1. *If* (π^*, ϕ_G^*) *is a solution to (19), there exists some* f *such that*

$$\phi_G^*(\theta_G) = \begin{cases} 0 & \text{if } \pi^*\theta_G < f \\ \pi^* & \text{if } \pi^*\theta_G \geq f; \end{cases} \tag{22}$$

2. *If* ϕ_P^* *is a solution to (20), there exists* p *such that*

$$\phi_P^*(\theta_P) = \begin{cases} 0 & \text{if } \theta_P < p \\ 1 & \text{if } \theta_P \geq p. \end{cases} \tag{23}$$

The proof relies on the fact that $\phi_J(\theta_J)$ appear linearly in both the objective functions and the constraints. As the argument is quite standard, we here only sketch the proof for Part 1. We take as granted that the problem can be solved by Lagrangian techniques (see Hellwig 2005 for justification). Let $\lambda \geq 0$ be the multiplier on the integral constraint (19b), $\gamma(\theta_G)$ be the multiplier on constraint $\phi_G(\theta_G) \geq 0$, and $\omega(\theta_G)$ be the multiplier on constraint $\pi - \phi(\theta_G) \geq 0$. The optimality condition for $\phi(\theta_G)$ then reads:

$$\theta_G + \lambda x_G(\theta_G) + \frac{\gamma(\theta_G) - \omega(\theta_G)}{h_G(\theta_G)} = 0, \tag{24}$$

together with the appropriate complementary slackness conditions for the non-negativity constraints. As $\theta_G + \lambda x_G(\theta_G)$ is continuous and strictly increasing under the regularity condition that $x_G(\cdot)$ is weakly increasing in θ_G, it follows from (24) that the solution has a threshold property, i.e., there exists some θ_G^* such that $\phi_G^*(\theta_G) = 0$ if $\theta_G < \theta_G^*$, and $\phi_G^*(\theta_G) = \pi^*$ if $\theta_G \geq \theta_G^*$, exactly as described by (22) if we let

[9] As a simple illustration, consider a case where there are two valuation types for the public good, θ_G^l and θ_G^h. Suppose that charging a flat fee equal to θ_G^l would violate the budget constraint, whereas charging θ_G^h would give a strict budget surplus. It is then obvious that welfare can be made higher by letting the low type agents consume the public good with some probability. The example can easily be extended to continuous densities.

$f = \theta_G^*/\pi^*$. Moreover, Lemma 1 implies that the threshold f in (22) is in fact closely associated transfer $t_G^*(\theta_G)$:

$$t_G^*(\theta_G) = \theta_G \phi_G^*(\theta_G) - \int_{\underline{\theta}_G}^{\theta_G} \phi_G^*(x)\,dx = \begin{cases} 0 & \text{if } \pi^*\theta_G < f \\ f & \text{if } \pi^*\theta_G \geq f. \end{cases}$$

Thus, the threshold f can be interpreted as an access fee for a lottery where consumers who pay f get access to the public good if it is provided, which happens with probability π^*.[10] The intuition for the proof of Part 2 is almost identical, with the threshold p interpreted as the price for the private good.

Lemmas 1 and 2 imply that, given the regularity conditions imposed, a solution to (16) may be obtained by solving the following simplified planning problem:

$$\max_{\{\pi,f,p\}} \pi \left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G} \theta_G dH_G(\theta_G) - K \right] + \int_p^{\overline{\theta}_P} (\theta_P - c)\, dH_P(\theta_P) \tag{25}$$

$$\text{s.t} \qquad 0 \leq f \left[1 - H_G\left(\frac{f}{\pi}\right) \right] - \pi K + (p-c)\left[1 - H_P(p)\right], \tag{26}$$

$$0 \leq \pi \leq 1. \tag{27}$$

Proposition 1 *Suppose that* $\mathrm{E}(\theta_G | \theta_G \geq 0) > K$ *and that* $\underline{\theta}_G < K$. *Then, in any optimal solution* (π^*, f^*, p^*) *to (25): (1)* $p^* > 0$; *(2)* $\pi^* > 0$; *(3)* $f^* > 0$.

Proof. First, write $\chi \equiv \frac{f}{\pi}$ and let λ, μ and γ respectively be the Lagrangian multiplier for the constraint (26) and the boundary constraints $\pi \geq 0$ and $\pi \leq 1$. The Kuhn-Tucker necessary conditions for an optimum are:

$$0 = \int_{\chi}^{\overline{\theta}_G} \theta_G dH_G(\theta_G) + (1+\lambda)\left[h_G(\chi)\chi^2 - K\right] + \mu - \gamma \tag{28}$$

$$0 = -\chi h_G(\chi) + \lambda\left[1 - H_G(\chi) - h_G(\chi)\chi\right] \tag{29}$$

$$0 = -(p-c)\, h_P(p) + \lambda\left[(1 - H_P(p)) - (p-c)\, h_P(p)\right] \tag{30}$$

$$0 = \lambda\left\{\pi\chi\left[1 - H_G(\chi)\right] + (p-c)\left[1 - H_P(p)\right] - K\pi\right\}, \lambda \geq 0 \tag{31}$$

$$\mu\pi = 0, \gamma(1-\pi) = 0, \mu \geq 0, \gamma \geq 0 \tag{32}$$

(Part 1): If $p^* < c$, then the first term on the right hand side in (30) is strictly positive and the second is weakly positive; thus the condition cannot hold. Suppose that $p^* = c$. Then, from (30) either $\lambda = 0$ or $1 - H_P(c) = 0$. Since the second condition is ruled out by the assumption that $c < \overline{\theta}_P$, the only possibility that remains is that $\lambda = 0$. But if $\lambda = 0$ at the optimal solution, then constraint (26) is not binding, which implies that χ^*, π^* must solve the following problem:

$$\max_{\{\chi,\pi\}} \pi \left[\int_{\chi}^{\overline{\theta}_G} \theta_G dH_G(\theta_G) - K \right]$$
$$\text{s.t.} \qquad 0 \leq \pi \leq 1.$$

If $\pi^* > 0$ in the solution to the above problem, then the objective function is monotonically decreasing in χ over $\left[\max\{\underline{\theta}_G, 0\}, \overline{\theta}_G\right]$; thus it must be that $\chi^* = \max\{\underline{\theta}_G, 0\}$. Thus, if $\pi^* > 0$, it

[10] Alternatively, the fee could be charged only when the good is provided, in which case the relevant access price would be $\frac{f}{\pi^*}$.

must maximize

$$\pi\left[\int_{\max\{\underline{\theta}_G,0\}}^{\bar{\theta}_G}\theta_G dH_G(\theta_G)-K\right]=\pi\left[\mathrm{E}\left(\theta_G|\theta_G\geq 0\right)-K\right].$$

By assumption $\mathrm{E}\left(\theta_G|\theta_G\geq 0\right)>K$, so the solution must be $\pi^*=1$, and thus $\chi^*=\max\left\{\underline{\theta}_G,0\right\}$. But, at $p^*=c,\chi^*=\max\left\{\underline{\theta}_G,0\right\}$ and $\pi^*=1$, the right hand side of the budget constraint (26) reads

$$\pi^*\left[\chi^*\left(1-H_G\left(\chi^*\right)\right)-K\right]+\left(p^*-c\right)\left[1-H_P\left(p^*\right)\right]=\max\left\{\underline{\theta}_G,0\right\}-K=\max\left\{\underline{\theta}_G-K,-K\right\},$$

which is strictly negative, thus the budget constraint (26) is violated, a contradiction. Hence, $p^*>c$ in any solution to (25).

(Part 2): As Part 1 establishes that $p^*>c$ in any optimum, it follows that there is a strict budget surplus if $\pi^*=0$ (the tax collected from the private goods due to $p^*>c$ is unspent). Instead, consider a positive public good provision probability π' as given by

$$\pi'=\frac{\left(p^*-c\right)\left[1-H_P\left(p^*\right)\right]}{K}>0.$$

By construction, constraint (26) is satisfied by the alternative simple mechanism $\left(\pi',\chi=0,p^*\right)$. Clearly $\left(\pi',\chi=0,p^*\right)$ improves the objective of (25) upon $\left(\pi^*=0,\chi=0,,p^*\right)$. A contradiction.

(Part 3): This is obvious if $\underline{\theta}_G>0$, since $\chi^*=\pi^*\underline{\theta}_G$ would be non-distortionary. Therefore, suppose $\chi^*=0$ and $\underline{\theta}_G\leq 0$. From condition (29), it follows that $\lambda\left[1-H_G(0)\right]=0$, which can only hold if $\lambda=0$. But, from the proof of Part 1, if $\lambda=0$, then $p^*=c$, which contradicts our conclusion in Part 1. ■

To summarize, under the standard regularity condition on the virtual valuation $x_J(\cdot)$, the best separate provision mechanism can be characterized by two prices, one for the public and one for the private good, and a provision probability for the public good. In particular, the sole government intervention in the private good market under the best separate provision mechanism can be interpreted as a "unit sales tax" in the amount of p^*-c. Otherwise, the operation of the private good market can be left completely to the private sector. In other words, in the best separate provision mechanism, the private good can be provided in a completely decentralized manner via a competitive market (subject to a sales tax). The only connection between the private and public goods is that the revenue from the unit tax on the private good is used as a cross-subsidy to partially fund the cost of the public good. This is not surprising, as the welfare loss from a small enough tax on the private good is of second order, whereas increasing the probability of public good provision or reducing access fees for the public good leads to a first-order welfare gain since we have under-provision of the public good at the constrained optimum. The intuition for the strictly positive public good user fee f^* is similar: excluding consumers with valuations just above zero leads to only a second order welfare loss, whereas the associated revenue from charging a positive f^* generates a first order welfare gain.

4 Publicly Provided Private Goods

In this section we extend the policy instruments for the government such that it is able to condition the provision probability and price for each of the two goods on the reported valuations of *both goods*. We interpret this as *public provision of both goods* since such joint provision mechanisms would not have been implementable if the private good were traded anonymously in the private sector.

For tractability we will study small perturbations of the best separate provision mechanism characterized in Proposition 1. Specifically, we will simply add a price τ, which is the fee charged to a consumer who consumes the bundle consisting of both the public and the private goods. Hence, we study mechanisms of the form (π, f, p, τ) instead of the mechanisms of the form (π, f, p) characterized in Proposition 1. Now, f is the user fee for the consumption of the public good only and p is the price for the private good charged to those who do not get access to the public good. If $\tau \neq f + p$, it requires that the government be actively involved in provision of the private good because such a scheme is feasible for the government only if it could monitor the consumers' purchases of the private goods. We will show that, generally, charging $\tau \neq f + p$ improves welfare.

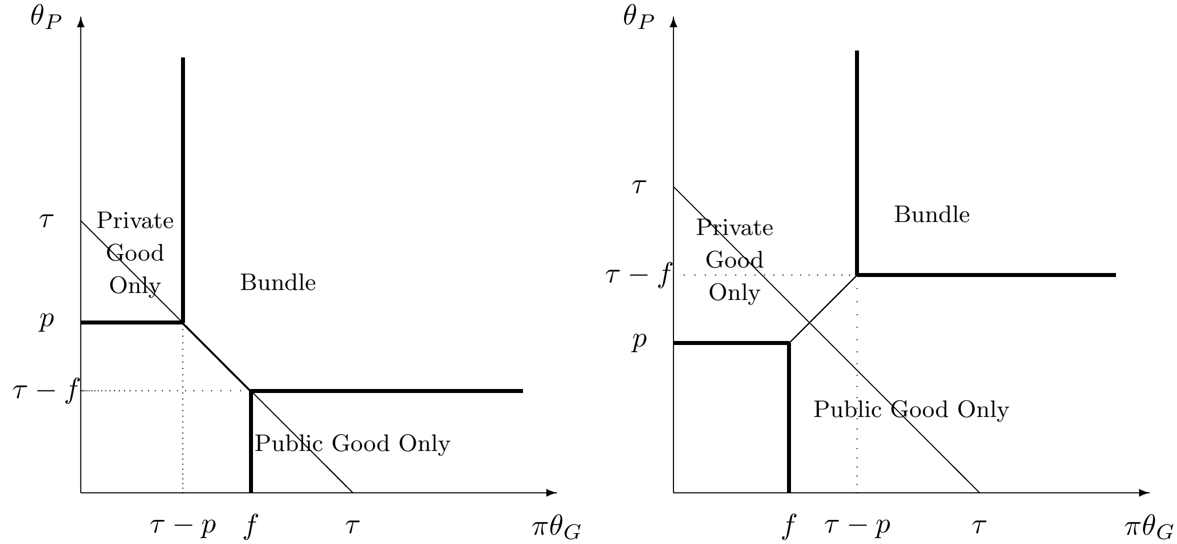
Our local argument is silent on what the optimal joint provision mechanism is. However, we are mainly interested in the *qualitative* question of whether public provision of a private good can be efficiency enhancing. As we show below that the perturbation outperforms the best separate provision mechanism we identified in Section 3, the constrained optimal joint provision mechanism must be one in which the government takes an active part in the provision of the private good.

One natural question is, does our analysis suggest that all private goods should be publicly provided in an optimal mechanism? *The answer is no.* A mechanism of the form (π, f, p, τ) with the feature $\tau \neq f + p$ could be implemented only if consumers *cannot engage in arbitrage.* Thus, our analysis suggests that only private goods for which consumers cannot engage in price arbitrage could be used by the government to improve efficiency. Indeed, this no-arbitrage restriction seems realistic for many goods that are publicly provided in the real world. For example, publicly-provided education (including public colleges), public health insurance, and public health care are all commodities that are difficult or impossible to resell, thus preventing arbitrage. The necessity to rule out consumer arbitrage strengthens our argument, as it provides an explanation for why some, but not all, private goods are publicly provided.

4.1 Demands, Budget Surplus and Social Surplus

As a first step, we need to calculate the social surplus as well as the budget surplus under simple pricing mechanisms of the form (π, f, p, τ). To do this we first derive the demands for the two goods and the bundle for a given allocation rule (π, f, p, τ).

Given (π, f, p, τ), a type-(θ_G, θ_P) consumer's demand is easy to characterize: (1) she will demand *only the public good* if $\pi\theta_G - f \geq 0,\ \pi\theta_G - f \geq \theta_P - p$, and $\pi\theta_G - f \geq \pi\theta_G + \theta_P - \tau$; (2) she will demand *only the private good* if $\theta_P - p \geq 0,\ \theta_P - p \geq \pi\theta_G - f$, and $\theta_P - p \geq \pi\theta_G + \theta_P - \tau$; (3) she will demand the bundle if $\pi\theta_G + \theta_P - \tau \geq 0,\ \pi\theta_G + \theta_P - \tau \geq \pi\theta_G - f$, and $\pi\theta_G + \theta_P - \tau \geq \theta_P - p$.



(a) Case $\tau \leq f + p$ (b) Case $\tau \geq f + p$

Figure 1: Consumer's Demand for Private Good Only, Public Good Only and the Bundle.

In each of the above three cases, which of inequalities are relevant depends on whether the bundle is cheaper or more expensive than the components. The two graphs in Figure 1 plot the consumer's demand for the private good and public goods as a function of her valuations (θ_G, θ_P) for the case $\tau \leq f + p$ and the case $\tau > f + p$ respectively. Notice that we have scaled down the valuation for the public good by the provision probability π in the graphs. Calculation based on Figure 1 immediately yields that the relevant demands can be summarized as in Table 1.

For notational simplicity, let $z = (\pi, f, p, \tau)$. Define by $G_1(z)$ the *budget surplus* (if positive) or budget deficit (if negative) given prices $z = (\pi, f, p, \tau)$ for the case $\tau \leq f + p$. Using the demands described in Table 1, we can write this as:

$$\begin{aligned} G_1(z) &= f\left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] + (p-c)\left[\int_{p}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] \\ &\quad + (\tau - c)\left[\int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}}\int_{\tau-\pi\theta_G}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G}\int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] - K\pi, \end{aligned} \tag{33}$$

Symmetrically, we let $G_2(z)$ denote the *budget surplus/deficit* given prices $z = (\pi, f, p, \tau)$ for the case $\tau \geq f + p$:

$$\begin{aligned} G_2(z) &= f\left[\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}}\int_{\underline{\theta}_P}^{\pi\theta_G+p-f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{p} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] \\ &\quad + (p-c)\left[\int_{p}^{\tau-f}\int_{\underline{\theta}_G}^{\frac{\theta_P+f-p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\tau-f}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] + (\tau-c)\left[\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G}\int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] - K\pi. \end{aligned} \tag{34}$$

Note that $G_1(z) = G_2(z)$ when $\tau = f + p$ which can be seen by substituting $\tau = f + p$ into (33) and (34).

	Case 1: $\tau \leq f + p$	Case 2: $\tau \geq f + p$
Public Good Only	$\int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}$	$\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G + p - f} h(\boldsymbol{\theta})\, d\theta + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}$
Private Good Only	$\int_{p}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}$	$\int_{p}^{\tau - f} \int_{\underline{\theta}_G}^{\frac{\theta_P + f - p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\tau - f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}$
Bundle	$\int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau - \pi\theta_G}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}$	$\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}$

Table 1: Summary of Demands.

Next, let $S_1(z)$ denote the *social surplus* given prices $z = (\pi, f, p, \tau)$ for the case $\tau \leq f + p$,

$$\begin{aligned} S_1(z) &= \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} \pi\theta_G h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{p}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} \\ &+ \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau - \pi\theta_G}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} - K\pi. \end{aligned} \tag{35}$$

Symmetrically, let $S_2(z)$ be the *social surplus* given prices $z = (\pi, f, p, \tau)$ for the case $\tau \geq f + p$:

$$\begin{aligned} S_2(z) &= \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G + p - f} \pi\theta_G h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} \pi\theta_G h(\boldsymbol{\theta})\, d\boldsymbol{\theta} \\ &+ \int_{p}^{\tau - f} \int_{\underline{\theta}_G}^{\frac{\theta_P + f - p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\tau - f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} \\ &+ \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} - K\pi. \end{aligned} \tag{36}$$

For the same reasons as that for the budget surplus, $S_1(z) = S_2(z)$ when $\tau = f + p$.

4.2 Two Auxiliary Problems

In order to provide conditions for when public provision of the private good outperforms the best decentralized outcome, we construct two *auxiliary* optimization problems. In words, the problems define the optimal price vectors (π, f, p, τ) under the restrictions that $\tau \leq f + p$ and $\tau \geq f + p$ respectively. Specifically, let (π^*, f^*, p^*) be the best separate provision mechanism characterized in Proposition 1; the two auxiliary problems are as follows. The first problem is:

$$\begin{aligned} &\max_{(f,p,\tau)} S_1(\pi^*, f, p, \tau) \\ \text{s.t.} \quad & G_1(\pi^*, f, p, \tau) \geq 0 \\ & f + p - \tau \geq 0. \end{aligned} \tag{37}$$

Problem (37) defines the best simple pricing policy in the form of (π, f, p, τ) under the restriction that $\pi = \pi^*$ and $\tau \leq f + p$. That is, the public good provision probability is fixed at the same level as in the best separate provision mechanism characterized in Proposition 1, and the bundle is restricted to be no more expensive than separate purchase of its components.

Similarly, the second auxiliary problem is:

$$\begin{aligned} & \max_{(f,p,\tau)} S_2(\pi^*, f, p, \tau) \\ \text{s.t.} \quad & G_2(\pi^*, f, p, \tau) \geq 0 \\ & \tau - f - p \geq 0. \end{aligned} \tag{38}$$

Problem (38) gives the best simple pricing policy in the form of (π, f, p, τ) under the restriction that $\pi = \pi^*$ and $\tau \geq f + p$. That is, the public good provision probability is fixed at the level as in the best separate provision mechanism characterized in Proposition 1, and the bundle is restricted to be no cheaper than separate purchase of its components.

It is important to note that $(f^*, p^*, f^* + p^*)$ is a feasible solution to both (37) and (38). It follows that a necessary condition for the best separate provision mechanism to be optimal when joint provision is feasible is that $(f^*, p^*, f^* + p^*)$ solves *both* (37) and (38). However, our main result provides a sufficient condition under which $(f^*, p^*, f^* + p^*)$ cannot solve both problems (37) and (38). Consequently, the constrained optimal joint provision mechanism must feature some element of public provision of the private good.

Differentiating (33) and evaluating at $z^* = (\pi^*, f^*, p^*, f^* + p^*)$, we find that:[11]

$$\begin{aligned} \frac{\partial G_1(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \{H_P(p^*|\theta_G) + (p^* - c) h_P(p^*|\theta_G)\} h_G(\theta_G) d\theta_G \\ &\quad - \frac{f^*}{\pi^*} H_P\left(p^*|\frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right) \end{aligned} \tag{39a}$$

$$\begin{aligned} \frac{\partial G_1(z^*)}{\partial p} &= \int_{\underline{\theta}_G}^{\frac{f^*}{\phi_G^*}} [\{(1 - H_P(p^*|\theta_G)) - (p^* - c) h_P(p^*|\theta_G)\}] h_G(\theta_G) d\theta_G \\ &\quad + \frac{f^*}{\phi_G^*}\left[1 - H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \end{aligned} \tag{39b}$$

$$\begin{aligned} \frac{\partial G_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\phi_G^*}}^{\overline{\theta}_G} \{(1 - H_P(p^*|\theta_G)) - (p^* - c) h_P(p^*|\theta_G)\} h_G(\theta_G) d\theta_G \\ &\quad - \frac{f^*}{\phi_G^*}\left[1 - H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \end{aligned} \tag{39c}$$

These expressions inform us about the effect on the budget when one slightly perturbs the relevant prices f, p and τ from $z^* = (\pi^*, f^*, p^*, f^* + p^*)$.

Likewise, if we differentiate $S_1(\cdot)$ in (35) and evaluate at $z = z^*$, we obtain:

$$\frac{\partial S_1(z^*)}{\partial f} = \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^* - c) h_P(p^*|\theta_G) h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} H_P\left(p^*|\frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right) \tag{40a}$$

$$\frac{\partial S_1(z^*)}{\partial p} = -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c) h_P(p^*|\theta_G) h_G(\theta_G) d\theta_G + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \tag{40b}$$

$$\frac{\partial S_1(z^*)}{\partial \tau} = -\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^* - c) h_P(p^*|\theta_G) h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right). \tag{40c}$$

We can also write out explicitly the gradients for S_2 and G_2, which we omit here.

[11] The details of the derivations for (39) and (40) are available in an appendix from the authors' website.

Our first preliminary result is that, evaluated at $z^* = (\pi^*, f^*, p^*, f^* + p^*)$, the partial derivatives of G_1 and G_2 are the same, and the partial derivatives of S_1 and S_2 also coincide. Letting $DG_i(z)$ and $DS_i(z)$ denote the gradient vectors for $i = 1, 2$, we thus have that:

Lemma 3 *$DG_1(z^*) = DG_2(z^*)$ and $DS_1(z^*) = DS_2(z^*)$*

Lemma 3 follows from straightforward but tedious algebra. It is analogous to the well-known "smooth pasting" condition in optimal control problem with switching points (see, e.g. Dixit 1993). Here, the relevant budget surplus function switch from G_1 to G_2 at z^*, and we have the analog of "value matching" of G_1 and G_2 at z^*, i.e., $G_1(z^*) = G_2(z^*)$. Lemma 3 simply states that the two functions are smoothly pasted at the switch point z^*. The same is true for the social surplus functions S_1 and S_2 at the switch point z^*.

Now we establish a useful lemma:

Lemma 4 *Let λ^* be the multiplier on constraint (26) corresponding to the solution (π^*, f^*, p^*) of problem (25). Also, let λ_i be the multiplier on the resource constraint $G_i(f, p, \tau; \pi^*)$ for $i = 1, 2$ in problem (37) and (38). Then,*

1. *$\lambda_1 = \lambda^*$ if z^* solves problem (37);*

2. *$\lambda_2 = \lambda^*$ if z^* solves problem (38).*

Proof. First consider (37). If z^* solves the problem, the Kuhn-Tucker necessary conditions for a solution must be fulfilled at z^*. Hence, there exists $\lambda_1 > 0$ and $\mu_1 \geq 0$ such that

$$\frac{\partial S_1(z^*)}{\partial f} + \lambda_1 \frac{\partial G_1(z^*)}{\partial f} + \mu_1 = 0 \tag{41a}$$

$$\frac{\partial S_1(z^*)}{\partial p} + \lambda_1 \frac{\partial G_1(z^*)}{\partial p} + \mu_1 = 0 \tag{41b}$$

$$\frac{\partial S_1(z^*)}{\partial \tau} + \lambda_1 \frac{\partial G_1(z^*)}{\partial \tau} - \mu_1 = 0 \tag{41c}$$

$$\mu_1 (f + p - \tau) = 0, \quad \mu_1 \geq 0 \tag{41d}$$

Using the expressions for the partial derivatives in (39) and (40), it is easy to check that:

$$\frac{\partial S_1(z^*)}{\partial f} + \frac{\partial S_1(z^*)}{\partial \tau} = -\frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right) \tag{42a}$$

$$\begin{aligned}\frac{\partial G_1(z^*)}{\partial f} + \frac{\partial G_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h_G(\theta_G)\, d\theta_G - \frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \left[1 - H_G\left(\frac{f^*}{\pi^*}\right)\right] - \frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right).\end{aligned} \tag{42b}$$

Combining (41a) and (41c), and using (42), we have that

$$-\frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right) + \lambda_1 \left\{\left[1 - H_G\left(\frac{f^*}{\pi^*}\right)\right] - \frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right)\right\} = 0. \tag{43}$$

This condition is the same as (29), one of the first order conditions to problem (25) when the goods are sold separately. It follows that $\lambda_1 = \lambda^*$, since otherwise (43) will be violated. This proves the first part.

For the second part, we note that the Kuhn-Tucker conditions for Problem (38) are

$$\begin{aligned}
\frac{\partial S_2(z^*)}{\partial f} + \lambda_2 \frac{\partial G_2(z^*)}{\partial f} - \mu_2 &= \frac{\partial S_1(z^*)}{\partial f} + \lambda_2 \frac{\partial G_1(z^*)}{\partial f} - \mu_2 = 0 \\
\frac{\partial S_2(z^*)}{\partial p} + \lambda_2 \frac{\partial G_2(z^*)}{\partial p} - \mu_2 &= \frac{\partial S_1(z^*)}{\partial p} + \lambda_2 \frac{\partial G_1(z^*)}{\partial p} - \mu_2 = 0 \\
\frac{\partial S_2(z^*)}{\partial \tau} + \lambda_2 \frac{\partial G_2(z^*)}{\partial \tau} + \mu_2 &= \frac{\partial S_1(z^*)}{\partial \tau} + \lambda_2 \frac{\partial G_1(z^*)}{\partial \tau} + \mu_2 = 0 \\
\mu_2 (f + p - \tau) &= 0, \mu_2 \geq 0
\end{aligned}$$

where the first equality in each line follows from Lemma 3. The same argument applies. ■

Together, Lemmas 3 and 4 make the Kuhn-Tucker conditions for problem (37) comparable with those for problem (38).

4.3 The Main Result

Now our main result about the sufficient condition under which public provision of private goods will improve welfare over the best separate provision mechanism follows:

Proposition 2 *Let λ^* be the multiplier on constraint (26) corresponding to a solution (π^*, f^*, p^*) of problem (25). Then, there exists a feasible simple pricing policy of the form (f, p, τ) that generates a higher social surplus than an optimal separate provision mechanism whenever*

$$DS_1(z^*) + \lambda^* DG_1(z^*) \neq 0. \tag{44}$$

Proof. Suppose to the contrary, $(f^*, p^*, f^* + p^*)$ solves both problems (37) and (38). Lemma 4 then implies that the multiplier in each problem must be given by λ^*. Thus if z^* is the best simple pricing policy for problem (37), then

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} + \mu_1 &= 0 \\
\frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} + \mu_1 &= 0 \\
\frac{\partial S_1(z^*)}{\partial \tau} + \lambda^* \frac{\partial G_1(z^*)}{\partial \tau} - \mu_1 &= 0 \\
\mu_1 (f + p - \tau) = 0, \qquad & \mu_1 \geq 0.
\end{aligned} \tag{45}$$

Similarly if z^* is the best simple pricing policy for problem (38), then by using Lemma 3, we have

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} - \mu_2 &= 0 \\
\frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} - \mu_2 &= 0 \\
\frac{\partial S_1(z^*)}{\partial \tau} + \lambda^* \frac{\partial G_1(z^*)}{\partial \tau} + \mu_2 &= 0 \\
\mu_2 (f + p - \tau) = 0, \qquad & \mu_2 \geq 0
\end{aligned} \tag{46}$$

Assume that $\mu_1 > 0$. Then, (45) implies that $\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} < 0$, which makes it impossible to find $\mu_2 \geq 0$ such that (46) holds. Symmetrically, if $\mu_2 > 0$, then $\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} > 0$, which makes it impossible to find $\mu_1 \geq 0$ such that (45) holds. Since z^* must solve both (37) and (38) for there to be no improvement we conclude that $\mu_1 = \mu_2 = 0$, or else there is some z better than z^*. The claim follows. ∎

Proposition 2 doesn't give us a concrete characterization of the optimal joint provision mechanism. However, since any joint provision mechanism must collect and use "private goods data" when allocating the public good it implies that the optimal mechanism has some public provision of the private good whenever condition (44) is satisfied.

4.4 Independence

Now we use Proposition 2 above to examine the case where θ_G and θ_P are independent. In this case, there is indeed always an improvement over the best separate provision policy.

Proposition 3 *Suppose that θ_G and θ_P are independent. Then*

$$\frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} > 0.$$

Proof. When $h_P(\theta_P|\theta_G) = h_P(\theta_P)$ for all θ_P, we have that

$$\begin{aligned} \frac{\partial G_1(z^*)}{\partial p} &= [1 - H_P(p^*) - (p^* - c) h_P(p^*)] H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\pi^*}[1 - H_P(p^*)] h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \frac{(p^* - c) h_P(p^*)}{\lambda^*} H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\pi^*}[1 - H_P(p^*)] h_G\left(\frac{f^*}{\pi^*}\right) \end{aligned}$$

where the second equality uses (30), the first order condition for p^* in the separate provision case. Next,

$$\frac{\partial S_1(z^*)}{\partial p} = -(p^* - c) h_P(p^*) H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\pi^*}[1 - H_P(p^*)] h_G\left(\frac{f^*}{\pi^*}\right)$$

Hence,

$$\begin{aligned} \frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} &= -(p^* - c) h_P(p^*) H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\pi^*}[1 - H_P(p^*)] h_G\left(\frac{f^*}{\pi^*}\right) \\ &\quad + \lambda^* \left\{\frac{(p^* - c) h_P(p^*)}{\lambda^*} H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\phi_G^*}[1 - H_P(p^*)] h_G\left(\frac{f^*}{\pi^*}\right)\right\} \\ &= (1 + \lambda^*) \frac{f^*}{\pi^*}[1 - H_P(p^*)] h_G\left(\frac{f^*}{\pi^*}\right) > 0. \quad \blacksquare \end{aligned}$$

Proposition 3 establishes that public provision of private goods is welfare improving in the important special case of independent valuations. By continuity, it also implies that there exists an *open set* of joint distribution functions for θ_G and θ_P for which public provision of private goods is efficiency enhancing.[12]

[12] We conjecture is that condition (44) is satisfied *generically* in the sense that it holds for almost all joint distribution functions.

5 Related Literature

Many papers in the literature interpret public provision of private goods as a political economy phenomenon. Epple and Romano (1996) consider a model where the level of a private good is determined by majority voting, and where the voters also decide if private supplements should be allowed. They show that a regime with positive government provision and no restriction on private supplement is majority preferred to one of either only market provision or only government provision under standard assumptions on voter preferences. The key intuition is that the median voter, who has income below the population mean, receives a positive net transfer if private goods are publicly provided. Fernandez and Rogerson (1995) use a similar model (with the crucial difference that the private good is only partially subsidized) to explain how public provision of private goods may occur even if the incidence favors wealthier households.

An alternative to these models based on majority voting is a literature that takes a normative perspective. Some of these papers take for granted that the transfers must be provided in-kind. For example, Besley and Coate (1991) considers a model where households may opt out from public provision and purchase a higher quality version of the good if they are dissatisfied with the quality of the publicly-provided good. If the willingness to pay for quality is increasing in wealth, mainly rich households opt out from the publicly provided private goods, implying that the system of public provision of private goods can serve as a transfer towards poor individuals.[13] Others, such as Blackorby and Donaldson (1988) construct models where in-kind transfers are superior to cash transfers because of its screening role, which allows better targeted transfers. In essence, the incentive to lie about health status to obtain a medical treatment is not as problematic as the incentive to lie to get some extra cash.[14] Blomquist and Christiansen (1995) and Blomquist, Christiansen and Micheletto (2008) introduce public provision of private good into an optimal income taxation framework and show that efficiency may be improved.

Limited commitment can also justify public provision of private goods. Coate (1995) analyzed an environment where the rich has altruistic preferences towards the poor and would like to insure the poor's income risks. If the poor are given cash, they may still opt not to purchase insurance to exploit the Samaritan's Dilemma. As a result, the rich may prefer to give the poor an in-kind transfer of insurance to overcome the difficulty of committing not to provide *ex post* assistance.[15] Yet another idea is to exploit indivisibilities. Garratt and Marshall (1994) considered the case of the public financing of college education. They argue that public financing of college education (a private good) provides gambles that families desire.

Hence, the literature has appealed to majority voting, sorting arguments, limited commitment and indivisibilities. However different these models are in terms of the mechanics, the desire to redistribute from rich to poor individuals is ultimately what drives them.

[13]Reated arguments based on a desire to redistribute income can be found in Blomquist and Christiansen (1999) and Cremer and Gahvarib (1997). Also, Gahvari and Mattos (2007) extends Besley and Coate (1991).

[14]See also Nichols and Zeckhauser (1982) for similar arguments.

[15]A similar model is considered by Bruce and Waldman (1991).

Our main contribution is to complement the literature above by pointing out that it is possible to justify public provision of private goods as a way of relaxing the incentive constraints facing a benevolent planner that needs to extract some revenue to correct a market failure. Most importantly, there is no need for the planner to have any redistributive preferences, which is the ultimate driving force in the existing literature. Hence, we need not interpret public provision of education or health care as a substitute for cash transfers to the poor. Instead, public provision may be an instrument to generate more efficient revenue extraction from the consumers in exactly the same way as commodity bundling may improve profits for a monopoly seller. We have illustrated this point in a very stylized model with a single public good and a single private good. However, to the extent that we believe that the public benefit and the cost of collecting the marginal tax dollar exceeds unity the logic should generalize.

At a more general level it has been known since Guesnerie and Roberts (1984) that asymmetric information can justify using quantity controls and prices simultaneously to improve sorting. The literature in commodity bundling considers very particular quantity controls where the marginal price for a good is made contingent on what other goods are consumed. Our paper shows that these well-known screening advantages of quantity controls/bundling can be translated into a novel theory for why private goods are publicly provided. We have chosen to illustrate this point in the simplest possible setup with unit demands, transferable utility and linear preferences in order to be able to obtain a clean characterization of the non-bundling benchmark. However, we see no particular reason why these simplifying assumptions should have any qualitative importance.

While not unique to our paper, it is important to note that our mechanism design approach avoids results that hinge on ad hoc restrictions on the set of feasible policies. In addition, like other papers based on sorting arguments we obtain a model where it is necessary to be able to stop consumer arbitrage. We view this as a desirable property as it provides some guidance about which private goods one should expect to be publicly provided.

How Our Problem Differs from McAffe et al. (1989)? Our expression $\partial G_1(z^*)/\partial p$ in (39b) is identical to the condition in Proposition 1 of McAfee *et al.* (1989). Expression (39a) can also be written in that form by reversing the roles of θ_G and θ_P. This is not a coincidence. The derivatives in (39a), (39b), and (39c) are the effects on profits given a marginal increase in f, p and τ respectively. In the case of McAfee *et al.* (1989), going from (39b) to their main result is relatively straightforward since they demonstrated that *profits* may be increased relative to separate pricing, by adding the bundle. Since they study profit maximization their analogue of (f^*, p^*) are chosen to solve a monopolist profit maximization problem under separate pricing. Thus their p^*, for example, must satisfy the first order optimality condition $[1 - H_P(p^*)] - (p^* - c)\, h_P(p^*) = 0$. If θ_G and θ_P

are independent, the first term in (39b) becomes:

$$\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \{[1 - H_P(p^* | \theta_G)] - (p^* - c) h_P(p^* | \theta_G)\} h_G(\theta_G) d\theta_G \tag{47}$$

$$= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \{[1 - H_P(p^*)] - (p^* - c) h_P(p^*)\} h_G(\theta_G),$$

which is equal to zero from the optimality condition of p^* in their problem. Thus it follows immediately from (39b) that a small increase in the price of the private good (or a small decrease in the price from the bundle) would increase the profits in the case of independence. It can also be verified that a small increase in the price of the public good also increases profits if θ_G and θ_P are stochastically independent.

Our problem differs from McAfee *et al.* (1989) in two respects. First of all, our goal is to demonstrate that bundling can increase *social welfare* rather than profits. Secondly, because (f^*, p^*) in our problem, as characterized in Proposition 1, are *not chosen to maximize profits*, we cannot use the first order conditions from the optimal separable provision mechanism in the same way as McAfee *et al.* (1989). Instead, we have to constrained optimization problems, which requires us to be able to link multiplier values across optimization problems. The latter is the main analytical difficulty relative McAfee *et al.* (1989).

6 Conclusion

This paper shows that public provision of private goods may be justified on pure efficiency grounds, even if the local government does not seek to redistribute resources. We believe that this is important for two reasons. Firstly, while the literature has identified situations where in-kind redistribution leads to improvements in how well targeted the transfers are, cash transfers are still superior to in-kind redistribution in many cases since the latter misallocates resources. Secondly, there are large scale programs that publicly provide private goods for which the redistributional effects are neutral or regressive. For these reasons we believe that a theory that does not rely upon a desire to redistribute is needed.

The theory developed in this paper is based on asymmetric information and the premise that governments also provide non-rival goods. In such an environment we show that public provision of a private good generates information that facilitates more efficient revenue extraction, which helps overcome inefficiencies in public good provision. Our main result establishes that public provision of the private good improves economic efficiency under a condition that is always fulfilled under independence and satisfied for a large set of joint distributions. Indeed, in an example we even showed that complete socialization of a private good may be better than market provision and optimal taxes.

The explanation of publicly provided private goods advanced in this paper differs substantially from explanations emphasized in the existing literature, which typically rely on preferences for redistribution. Unlike most of the literature, we are also able to avoid ad hoc restrictions on the set of policy instruments, such as ruling out cash transfers. In contrast, we take a purist mechanism design perspective, so that the benchmark outcome without public provision of the private good is constrained optimal subject to incentive feasibility.

Finally, our analysis also exemplifies a more general point about the optimal taxation literature. Our model combines optimal commodity taxation with a decision on how to provide an excludable public good. We found that the marginal price for access to the public good for consumers that purchase the private good should be different from those who do not. Hence, our model is a stylized example where provision and user fees for the public good, and taxes on the private good must be jointly determined in order to achieve economic efficiency. Our paper thus illustrates that the standard practice of separating the question of how a given budget should be spent from the question of how a given tax revenue should be raised generates efficiency losses.

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A Details About Lemma 1.

Let $(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*)$ solve the planning problem (16). Because, as argued in (17), the incentive compability constraints (16b) can be separated for the private good and the public good, we know that, fix (ϕ_P^*, t_P^*), (π^*, ϕ_G^*, t_G^*) must solve:

$$\max_{(\pi,\phi_G,t_G)} \int_{\Theta} \left[\phi_G(\theta_G)\theta_G - t_G(\theta_G)\right] dH_G(\theta_G) \tag{A1}$$

$$\text{s.t.} \quad 0 \le \phi_G(\theta_G)\theta_G - t_G(\theta_G) - \phi_G(\widehat{\theta}_G)\theta_G + t_G(\widehat{\theta}_G), \quad \forall \theta_G, \widehat{\theta}_G \in \Theta_G, \tag{A2}$$

$$0 \le \phi_G(\theta_G)\theta_G - t_G(\theta_G) + \phi_P^*(\theta_P)\theta_P - t_P^*(\theta_P), \quad \forall (\theta_G, \theta_P) \in \Theta_G \times \Theta_P,$$

$$0 \le \int_{\Theta_G} t_G(\theta_G)\, dH_G(\theta_G) - K\pi + \underbrace{\int_{\Theta_P} t_P^*(\theta_P)\, dH_P(\theta_P) - \int_{\Theta_P} c\phi_P^*(\theta_P)\, dH_P(\theta_P)}_{\text{constant}} \tag{A3}$$

$$0 \le \phi_G(\theta_G) \le \pi.$$

The optimization problem (A1) takes the private good allocation rule (ϕ_P^*, t_P^*) from an optimal mechanism as given and solves for an optimal allocation of the public good conditional on the transfer between markets (the constant in A3) and the reservation utilities implied by (ϕ_P^*, t_P^*).

Symmetrically, fix (π^*, ϕ_G^*, t_G^*), (ϕ_P^*, t_P^*) must solve:

$$\max_{(\phi_P,t_P)} \int_{\Theta_P} \left[\phi_P(\theta_P)\theta_P - t_P(\theta_P)\right] dH_P(\theta_P) \tag{A4}$$

$$\text{s.t.} \quad 0 \le \phi_P(\theta_P)\theta_P - t_P(\theta_P) - \phi_P(\widehat{\theta}_P)\theta_P + t_P(\widehat{\theta}_P) \quad \forall \theta_P, \widehat{\theta}_P \in \Theta_P \tag{A5}$$

$$0 \le \phi_G^*(\theta_G)\theta_G - t_G^*(\theta_G) + \phi_P(\theta_P)\theta_P - t_P(\theta_P) \quad \forall (\theta_G, \theta_P) \in \Theta_G \times \Theta_P$$

$$0 \le \underbrace{\int_{\Theta_G} t_G^*(\theta_G)\, dH_G(\theta_G) - K\pi^*}_{\text{constant}} + \int_{\Theta_P} t_P(\theta_P)\, dH_P(\theta_P) - \int_{\Theta_P} c\phi_P(\theta_P)\, dH_P(\theta_P).$$

That is, taking the public good provision as given, (ϕ_P^*, t_P^*) solves for the least distorted allocation of private goods conditional on the transfer to the other market and reservation utilities implied.

Define the "indirect utility functions"

$$U_J(\theta_J) \equiv \theta_J \phi_J(\theta_J) - t_J(\theta_J), \tag{A6}$$

for $J = G, P$. A routine argument based on reasoning akin to the envelope theorem (see, e.g. Myerson 1981 or Mas-Colell *et al.* 1995, page 888) can be used to establish:

Lemma A1 *Suppose that the marginal density $h_J(\theta_J)$ is strictly positive on its support $\Theta_J = [\underline{\theta_J}, \overline{\theta_J}]$. Then, (ϕ_J, t_J) satisfies the incentive compatibility constraints in (A2) and (A5) respectively if and only if ϕ_J is weakly increasing in θ_J and*

$$U_J(\theta_J) = U_J(\widehat{\theta}_J) + \int_{\widehat{\theta}_J}^{\theta_J} \phi_J(x)\, dx \quad \forall \theta_J, \widehat{\theta}_J \in \Theta_J.$$

Equally routine procedures using the characterization in Lemma A1 show that the aggregate transfer revenues from the public goods fees and the private goods fees respectively can be determined uniquely from the utility of the lowest type and the provision rules.

Lemma A2 *Suppose that* (π, ϕ_G, t_G) *and* (ϕ_P, t_P) *satisfy the incentive compatibility constraints in (A2) and (A5) receptively if and only* ϕ_J *is weakly increasing in* θ_J *and*

$$\int_{\Theta_J} t_J(\theta_J)\, dH_J(\theta_J) = \int_{\Theta_J} \phi_J(\theta_J)\, x_J(\theta_J)\, dH_J(\theta_J) - U_J(\underline{\theta_J}).$$

Furthermore, it is without loss of generality assume that the participation constraint of type $(\underline{\theta}_G, \underline{\theta}_P)$ binds, so that

$$\phi^*(\underline{\theta}_G)\,\underline{\theta}_G - t_G^*(\underline{\theta}_G) + \phi_P^*(\underline{\theta}_P)\,\underline{\theta}_P - t_P^*(\underline{\theta}_P) = 0,$$

at a solution to problem (16). Higher types can mimic $(\underline{\theta}_G, \underline{\theta}_P)$, so, just like in unidimensional problems, incentive compatibility automatically implies that the participation constraints hold for higher types, provided that it is satisfied for the lowest type.

Lemma A2 together with the binding participation constraint for $(\underline{\theta}_G, \underline{\theta}_P)$ suggests a reformulation of the public good problem (A1) as Problem (19). Specifically, Problem (19) is derived from problem (A1) as follows: (1) the transfers are eliminated from the objective function by substitution from the feasibility constraint (A3), which is assumed to bind (all constants have been eliminated); (2) All redundant participation constraints have been eliminated.

Problem (20) is derived from problem (A4) analogously.