For Referees Only, Not Intended for Publication.

B Appendix: Omitted Details of Some Derivations

B.1 Derivation of Derivatives in (39):

For simplicity of notation, define:

$$\begin{aligned} A_1(z;\phi_G) &\equiv f\left[\int_{\frac{f}{\phi_G}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\,d\boldsymbol{\theta}\right], \\ B_1(z;\phi_G) &\equiv (p-c)\left[\int_{p}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{\tau-p}{\phi_G^*}} h(\boldsymbol{\theta})\,d\boldsymbol{\theta}\right], \\ C_1(z;\phi_G) &\equiv (\tau-c)\left[\int_{\frac{\tau-p}{\phi_G}}^{\frac{f}{\phi_G}}\int_{\tau-\phi_G\theta_G}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} + \int_{\frac{f}{\phi_G}}^{\overline{\theta}_G}\int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta}\right], \end{aligned}$$

so that

$$G_1(z;\phi_G) = A_1(z;\phi_G) + B_1(z;\phi_G) + C_1(z;\phi_G) - K\phi_G.$$

Differentiating with respect to f, we have:

$$\begin{aligned} \frac{\partial A_1(z)}{\partial f} &= \int_{\frac{f}{\pi}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - f\left[\int_{\underline{\theta}_P}^{\tau-f} h\left(\frac{f}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} h(\theta_G,\tau-f)\,d\theta_G\right], \\ \frac{\partial B_1(z)}{\partial f} &= 0, \\ \frac{\partial C_1(z)}{\partial f} &= (\tau-c)\left[\int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{f}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P - \int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{f}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} h(\theta_G,\tau-f)\,d\theta_G\right] \\ &= (\tau-c)\left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G} h(\theta_G,\tau-f)\,d\theta_G\right]. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\partial A_1(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - f^*\left[\int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi},\theta_P\right)\frac{1}{\pi^*}d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right], \\ \frac{\partial B_1(z^*)}{\partial f} &= 0, \\ \frac{\partial C_1(z^*)}{\partial f} &= (f^*+p^*-c)\left[\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right]. \end{aligned}$$

So,

$$\frac{\partial G_1(z^*)}{\partial f} = \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta})\,d\theta_P + (p^*-c)\,h(\theta_G,p^*)\right\}d\theta_G - f^*\int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P \tag{B7}$$

$$= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\int_{\underline{\theta}_P}^{p^*} \underbrace{\frac{h(\boldsymbol{\theta})}{\int_{\underline{\theta}_P}^{\overline{\theta}_P} h(\theta_G,\theta_P)\,d\theta_P}}_{h_P(\theta_P|\theta_G)}d\theta_P + (p^*-c)\underbrace{\frac{h(\theta_G,p^*)}{\int_{\underline{\theta}_P}^{\overline{\theta}_P} h(\theta_G,\theta_P)\,d\theta_P}}_{h_P(p^*|\theta_G)}\right\}\underbrace{\left(\int_{\underline{\theta}_P}^{\overline{\theta}_P} h(\theta_G,\theta_P)\,d\theta_P\right)}_{h_G(\theta_G)}d\theta_G d\theta_G$$

$$-f^*\int_{\underline{\theta}_P}^{p^*}\underbrace{\frac{h\left(\frac{f^*}{\pi^*},\theta_P\right)}{\int_{\underline{\theta}_P}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)d\theta_P}}_{h_P\left(\theta_P|\frac{f^*}{\pi^*}\right)}\frac{1}{\pi^*}d\theta_P\underbrace{\int_{\underline{\theta}_P}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)d\theta_P}_{h_G\left(\frac{f^*}{\pi^*}\right)}$$

$$= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\int_{\underline{\theta}_P}^{p^*} h_P(\theta_P|\theta_G)\,d\theta_P + (p^*-c)\,h_P(p^*|\,\theta_G)\right\}h_G(\theta_G)\,d\theta_G - \frac{f^*}{\pi^*}\int_{\underline{\theta}_P}^{p^*} h_P\left(\theta_P|\frac{f^*}{\pi^*}\right)d\theta_P h_G\left(\frac{f^*}{\pi^*}\right)$$

$$= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\{H_P(p^*|\,\theta_G) + (p^*-c)\,h_P(p^*|\,\theta_G)\}\,h_G(\theta_G)\,d\theta_G - \frac{f^*}{\pi^*}H_P\left(p^*|\frac{f^*}{\pi^*}\right)h_G\left(\frac{f^*}{\pi^*}\right).$$

Differentiating with respect to τ yields:

$$\frac{dA_1(z)}{d\tau} = f\left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G} h(\theta_G,\tau-f)\,d\theta_G\right]$$

$$\frac{\partial B_1(z)}{\partial\tau} = (p-c)\left[\int_p^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P\right]$$

$$\frac{\partial C_1(z)}{\partial\tau} = \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}}\int_{\tau-\pi\theta_G}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G}\int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta}$$
$$-(\tau-c)\left[\int_p^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} h(\theta_G,\tau-\pi\theta_G)\,d\theta_G\right]$$
$$-(\tau-c)\left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G} h(\theta_G,\tau-f)\,d\theta_G\right]$$

Hence,

$$\frac{dA_1(z^*)}{d\tau} = f^*\left[\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,\tau-f^*)\,d\theta_G\right]$$

$$\frac{\partial B_1(z^*)}{\partial\tau} = (p^*-c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right]$$

$$\frac{\partial C_1(z^*)}{\partial\tau} = \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - (p^*+f^*-c)\left\{\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right] + \left[\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right]\right\}.$$

It follows that

$$\begin{aligned}
\frac{\partial G_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - f^*\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right] - (p^*-c)\left[\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\boldsymbol{\theta}\right]\\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\theta_P - (p^*-c)\,h(\theta_G,p^*)\right\}d\theta_G - f^*\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right]\\
\text{/same steps as in (B7)/} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\left[1-H_P(p^*|\,\theta_G)\right]-(p^*-c)\,h_P(p^*|\,\theta_G)\right\}h_G(\theta_G)\,d\theta_G - \frac{f^*}{\pi^*}\left[1-H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right]h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned}$$

Differentiating with respect to p yields:

$$\begin{aligned}
\frac{\partial A_1(z)}{\partial p} &= 0,\\
\frac{\partial B_1(z)}{\partial p} &= \int_{p}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - (p-c)\left[\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta_G,p)\,d\theta_G + \int_{p}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P\right],\\
\frac{\partial C_1(z)}{\partial p} &= (\tau-c)\left[\int_{p}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial A_1(z^*)}{\partial p} &= 0,\\
\frac{\partial B_1(z^*)}{\partial p} &= \int_{p^*}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - (p^*-c)\left[\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\theta_G,p^*)\,d\theta_G + \int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right],\\
\frac{\partial C_1(z^*)}{\partial p} &= (f^*+p^*-c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right],
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial G_1(z^*)}{\partial p} &= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}}\left[\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\theta_P - (p^*-c)\,h(\theta_G,p^*)\right]d\theta_G + f^*\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right]\\
\text{/same steps as in (B7)/} &= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}}\left[\left\{\left(1-H_P(p^*|\,\theta_G)\right)-(p^*-c)\,h_P(p^*|\,\theta_G)\right\}\right]h_G(\theta_G)\,d\theta_G\\
&\quad + \frac{f^*}{\pi^*}\left[1-H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right]h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned}$$

B.2 Derivation of Derivatives in (40):

Let

$$\begin{aligned}
D_1(z) &= \int_{\frac{f}{\pi}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{\tau-f} \pi\theta_G h(\boldsymbol{\theta})\,d\boldsymbol{\theta},\\
E_1(z) &= \int_{p}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P-c)\,h(\boldsymbol{\theta})\,d\boldsymbol{\theta},\\
H_1(z) &= \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}}\int_{\tau-\pi\theta_G}^{\overline{\theta}_P} (\pi\theta_G+\theta_P-c)\,h(\boldsymbol{\theta})\,d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G}\int_{\tau-f}^{\overline{\theta}_P} (\pi\theta_G+\theta_P-c)\,h(\boldsymbol{\theta})\,d\boldsymbol{\theta}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial D_1(z)}{\partial f} &= -\int_{\underline{\theta}_P}^{\tau-f} fh\left(\frac{f}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P - \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \pi\theta_G h(\theta_G,\tau-f)\,d\theta_G \\
\frac{\partial D_1(z)}{\partial p} &= 0 \\
\frac{\partial D_1(z)}{\partial \tau} &= \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \pi\theta_G h(\theta_G,\tau-f)\,d\theta_G
\end{aligned}$$

and thus,

$$\begin{aligned}
\frac{\partial D_1(z^*)}{\partial f} &= -\int_{\underline{\theta}_P}^{p^*} f^* h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \pi^*\theta_G h(\theta_G,p^*)\,d\theta_G \qquad \text{(B8)} \\
\frac{\partial D_1(z^*)}{\partial p} &= 0 \\
\frac{\partial D_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \pi^*\theta_G h(\theta_G,p^*)\,d\boldsymbol{\theta}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial E_1(z)}{\partial f} &= 0, \\
\frac{\partial E_1(z)}{\partial p} &= -\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (p-c)\,h(\theta_G,p)\,d\theta_G - \int_{p}^{\overline{\theta}_P} (\theta_P-c)\,h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P, \\
\frac{\partial E_1(z)}{\partial \tau} &= \int_{p}^{\overline{\theta}_P} (\theta_P-c)\,h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P,
\end{aligned}$$

thus,

$$\begin{aligned}
\frac{\partial E_1(z^*)}{\partial f} &= 0 \qquad \text{(B9)} \\
\frac{\partial E_1(z^*)}{\partial p} &= -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^*-c)\,h(\theta_G,p^*)\,d\theta_G - \int_{p^*}^{\overline{\theta}_P} (\theta_P-c)\,h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P \\
\frac{\partial E_1(z^*)}{\partial \tau} &= \int_{p^*}^{\overline{\theta}_P} (\theta_P-c)\,h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P.
\end{aligned}$$

Finally,

$$\begin{aligned}
\frac{\partial H_1(z)}{\partial f} &= \int_{\tau-f}^{\overline{\theta}_P} (f+\theta_P-c)\, h\left(\frac{f}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P - \int_{\tau-f}^{\overline{\theta}_P} (f+\theta_P-c)\, h\left(\frac{f}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P \\
&\quad + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} (\pi\theta_G+\tau-f-c)\, h(\theta_G,\tau-f)\, d\theta_G \\
&= \int_{\frac{f}{\pi}}^{\overline{\theta}_G} (\pi\theta_G+\tau-f-c)\, h(\theta_G,\tau-f) \\
\frac{\partial H_1(z)}{\partial p} &= \int_{p}^{\overline{\theta}_P} (\tau-p+\theta_P-c)\, h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P \\
\frac{\partial H_1(z)}{\partial \tau} &= -\int_{p}^{\overline{\theta}_P} (\tau-p+\theta_P-c)\, h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P - \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} (\tau-c)\, h(\theta_G,\tau-\pi\theta_G)\, d\theta_G \\
&\quad - \int_{\frac{f}{\pi}}^{\overline{\theta}_G} (\pi\theta_G+\tau-f-c)\, h(\theta_G,\tau-f)\, d\theta_G
\end{aligned}$$

thus,

$$\begin{aligned}
\frac{\partial H_1(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (\pi^*\theta_G+p^*-c)\, h(\theta_G,p^*)\, d\theta_G \qquad \text{(B10)} \\
\frac{\partial H_1(z^*)}{\partial p} &= \int_{p^*}^{\overline{\theta}_P} (f^*+\theta_P-c)\, h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P \\
\frac{\partial H_1(z^*)}{\partial \tau} &= -\int_{p^*}^{\overline{\theta}_P} (f^*+\theta_P-c)\, h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (\pi^*\theta_G+p^*-c)\, h(\theta_G,p^*)\, d\theta_G.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial f} &= \frac{\partial D_1(z^*)}{\partial f} + \frac{\partial E_1(z^*)}{\partial f} + \frac{\partial H_1(z^*)}{\partial f} \\
&= -\int_{\underline{\theta}_P}^{p^*} f^* h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \pi^*\theta_G h(\theta_G,p^*)\, d\theta_G + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (\pi^*\theta_G+p^*-c)\, h(\theta_G,p^*)\, d\theta_G \\
&= -\int_{\underline{\theta}_P}^{p^*} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*},\theta_P\right) d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^*-c)\, h(\theta_G,p^*)\, d\theta_G \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^*-c)\, h_P(p^*|\,\theta_G)\, h_G(\theta_G)\, d\theta_G - \frac{f^*}{\pi^*} H_P\left(p^*|\frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right)
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial p} &= \frac{\partial D_1(z^*)}{\partial p} + \frac{\partial E_1(z^*)}{\partial p} + \frac{\partial H_1(z^*)}{\partial p} \\
&= -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^*-c)\, h(\theta_G,p^*)\, d\theta_G - \int_{p^*}^{\overline{\theta}_P} (\theta_P-c)\, h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P \\
&\quad + \int_{p^*}^{\overline{\theta}_P} (f^*+\theta_P-c)\, h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P \\
&= -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^*-c)\, h(\theta_G,p^*)\, d\theta_G + \int_{p^*}^{\overline{\theta}_P} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*},\theta_P\right) d\theta_P \\
&= -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^*-c)\, h_P(p^*|\,\theta_G)\, h_G(\theta_G)\, d\theta_G + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right),
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial \tau} &= \frac{\partial D_1(z^*)}{\partial \tau} + \frac{\partial E_1(z^*)}{\partial \tau} + \frac{\partial H_1(z^*)}{\partial \tau} \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \pi^* \theta_G h(\theta_G, p^*)\, d\boldsymbol{\theta} + \int_{p^*}^{\overline{\theta}_P} (\theta_P - c)\, h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
&\quad - \int_{p^*}^{\overline{\theta}_P} (f^* + \theta_P - c)\, h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (\pi^* \theta_G + p^* - c)\, h(\theta_G, p^*)\, d\theta_G \\
&= -\int_{p^*}^{\overline{\theta}_P} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^* - c)\, h(\theta_G, p^*)\, d\theta_G \\
&= -\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^* - c)\, h_P(p^* | \theta_G)\, h_G(\theta_G)\, d\theta_G - \frac{f^*}{\pi^*}\left[1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned}$$

B.3 Calculations in Proving Lemma 3

- **Part 1:** $DG_1(z^*) = DG_2(z^*)$:

Write $G_2(z) = A_2(z) + B_2(z) + C_2(z) - K\pi$ where:

$$\begin{aligned}
A_2(z) &\equiv f\left[\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G + p - f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] \\
B_2(z) &\equiv (p - c)\left[\int_{p}^{\tau - f} \int_{\underline{\theta}_G}^{\frac{\theta_P + f - p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\tau - f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] \\
C_2(z) &\equiv (\tau - c)\left[\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right].
\end{aligned}$$

Differentiating A_2 we get:

$$\begin{aligned}
\frac{\partial A_2(z)}{\partial f} &= \left[\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G + p - f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} h(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right] \\
&\quad - f\left[\int_{\underline{\theta}_P}^{p} h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} h(\theta_G, \pi\theta_G + p - f)\, d\theta_G + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} h(\theta_G, \tau - f)\, d\theta_G\right] \\
\frac{\partial A_2(z)}{\partial p} &= -f\int_{\underline{\theta}_P}^{\tau - f} h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + f\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} h(\theta_G, \pi\theta_G + p - f)\, d\theta_G + f\int_{\underline{\theta}_P}^{\tau - f} h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&= f\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} h(\theta_G, \pi\theta_G + p - f)\, d\theta_G \\
\frac{\partial A_2(z)}{\partial \tau} &= f\int_{\underline{\theta}_P}^{\tau - f} h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - f\int_{\underline{\theta}_P}^{\tau - f} h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + f\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} h(\theta_G, \tau - f)\, d\theta_G \\
&= f\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} h(\theta_G, \tau - f)\, d
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial A_2\left(z^*;\pi\right)}{\partial f} &= \int_{\frac{f^*}{\pi}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{p^*} h\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta} - f^*\left[\int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\frac{f^*}{\pi}}^{\overline{\theta}_G} h\left(\theta_G,p^*\right)d\theta_G\right]\\
\frac{\partial A_2\left(z^*;\pi\right)}{\partial p} &= 0\\
\frac{\partial A_2\left(z^*;\pi\right)}{\partial \tau} &= f^*\int_{\frac{f^*}{\pi}}^{\overline{\theta}_G} h\left(\theta_G,p^*\right)d\theta_G.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial B_2\left(z\right)}{\partial f} &= \left(p-c\right)\left[-\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h\left(\theta_G,\tau-f\right)d\theta_G + \int_{p}^{\tau-f} h\left(\frac{\theta_P+f-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h\left(\theta_G,\tau-f\right)d\theta_G\right]\\
&= \left(p-c\right)\left[\int_{p}^{\tau-f} h\left(\frac{\theta_P+f-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P\right]\\
\frac{\partial B_2\left(z\right)}{\partial p} &= \int_{p}^{\tau-f}\int_{\underline{\theta}_G}^{\frac{\theta_P+f-p}{\pi}} h\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta} + \int_{\tau-f}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta}\\
&\quad -\left(p-c\right)\left[\int_{\underline{\theta}_G}^{\frac{f}{\pi}} h\left(\theta_G,p\right)d\theta_G + \int_{p}^{\tau-f} h\left(\frac{\theta_P+f-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)d\theta_P\right]\\
\frac{\partial B_2\left(z\right)}{\partial \tau} &= \left(p-c\right)\left[\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h\left(\theta_G,\tau-f\right)d\boldsymbol{\theta} - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h\left(\theta_G,\tau-f\right)d\theta_G + \int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_G\right]\\
&= \left(p-c\right)\left[\int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_G\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial B_2\left(z^*\right)}{\partial f} &= 0\\
\frac{\partial B_2\left(z^*\right)}{\partial p} &= \int_{p^*}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta} - \left(p^*-c\right)\left[\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h\left(\theta_G,p^*\right)d\theta_G + \int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)d\theta_P\right]\\
\frac{\partial B_2\left(z^*\right)}{\partial \tau} &= \left(p^*-c\right)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_G\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial C_2\left(z\right)}{\partial f} &= \left(\tau-c\right)\left[\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} h\left(\theta_G,\tau-f\right)d\theta_G\right]\\
\frac{\partial C_2\left(z\right)}{\partial p} &= \left(\tau-c\right)\left[\int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P\right]\\
\frac{\partial C_2\left(z\right)}{\partial \tau} &= \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G}\int_{\tau-f}^{\overline{\theta}_P} h\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta} - \left(\tau-c\right)\left[\int_{\tau-f}^{\overline{\theta}_P} h\left(\frac{\tau-p}{\pi},\theta_P\right)\frac{1}{\pi}d\theta_P + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} h\left(\theta_G,\tau-f\right)d\theta_G\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial C_2(z^*)}{\partial f} &= (f^*+p^*-c)\left[\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right] \\
\frac{\partial C_2(z^*)}{\partial p} &= (f^*+p^*-c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right] \\
\frac{\partial C_2(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - (f^*+p^*-c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right]
\end{aligned}$$

Combining terms we get that

$$\begin{aligned}
\frac{\partial G_2(z^*)}{\partial f} &= \frac{\partial A_2(z^*)}{\partial f} + \frac{\partial B_2(z^*)}{\partial f} + \frac{\partial C_2(z^*)}{\partial f} \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - f^*\left[\int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right] \\
&\quad + (f^*+p^*-c)\left[\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G,p^*)\,d\theta_G\right] \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta})\,d\theta_P + (p^*-c)\,h(\theta_G,p^*)\right\}d\theta_G - f^*\int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{\int_{\underline{\theta}_P}^{p^*} h_P(\theta_P|\theta_G)\,d\theta_P + (p^*-c)\,h_P(p^*|\,\theta_G)\right\}h_G(\theta_G)\,d\theta_G - \frac{f^*}{\pi^*}\int_{\underline{\theta}_P}^{p^*} h_P\left(\theta_P|\frac{f^*}{\pi^*}\right)d\theta_P h_G\left(\frac{f^*}{\pi^*}\right) \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G}\left\{H_P(p^*|\,\theta_G) + (p^*-c)\,h_P(p^*|\,\theta_G)\right\}h_G(\theta_G)\,d\theta_G - \frac{f^*}{\pi^*}H_P\left(p^*|\frac{f^*}{\pi^*}\right)h_G\left(\frac{f^*}{\pi^*}\right) \\
&= \frac{\partial G_1(z^*)}{\partial f},
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial G_2(z^*)}{\partial p} &= \frac{\partial A_2(z^*)}{\partial p} + \frac{\partial B_2(z^*)}{\partial p} + \frac{\partial C_2(z^*)}{\partial p} \\
&= \int_{p^*}^{\overline{\theta}_P}\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} - (p^*-c)\left[\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\theta_G,p^*)\,d\theta_G + \int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)d\theta_P\right] \\
&\quad + (f^*+p^*-c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right] \\
&= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}}\left\{\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\theta_P - (p^*-c)\,h(\theta_G,p^*)\right\}d\theta_G + f^*\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*},\theta_P\right)\frac{1}{\pi^*}d\theta_P\right] \\
&= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}}\left\{1 - H_P(p^*|\,\theta_G) - (p^*-c)\,h_P(p^*|\,\theta_G)\right\}h_G(\theta_G)\,d\theta_G + f^*\left[1 - H_P\left(p^*|\frac{f^*}{\pi^*}\right)\right]h_G\left(\frac{f^*}{\pi^*}\right) \\
&= \frac{\partial G_1(z^*)}{\partial p}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial G_2(z^*)}{\partial \tau} &= \frac{\partial A_2(z^*)}{\partial \tau} + \frac{\partial B_2(z^*)}{\partial \tau} + \frac{\partial C_2(z^*)}{\partial \tau} \\
&= f^* \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G, p^*)\, d\theta_G + (p^* - c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_G\right] \\
&\quad + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\boldsymbol{\theta} - (f^* + p^* - c)\left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h(\theta_G, p^*)\, d\theta_G\right] \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \left\{\int_{p^*}^{\overline{\theta}_P} h(\boldsymbol{\theta})\, d\theta_P - (p^* - c)\, h(\theta_G, p^*)\right\} d\theta_G - f^* \left[\int_{p^*}^{\overline{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P\right] \\
&= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \left\{1 - H_P(p^* | \theta_G) - (p^* - c)\, h_P(p^* | \theta_G)\right\} h_G(\theta_G)\, d\theta_G - \frac{f^*}{\pi^*}\left[1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right)\right] \\
&= \frac{\partial G_1(z^*)}{\partial \tau}
\end{aligned}
$$

- **Part 2: $DS_1(z^*) = DS_2(z^*)$:**

Write $S_2(z) = D_2(z) + E_2(z) + F_2(z) - K\pi$ where

$$
\begin{aligned}
D_2(z) &= \int_{\frac{f}{\pi}}^{\frac{\tau - p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G + p - f} \pi\theta_G h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\frac{\tau - p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} \pi\theta_G h(\boldsymbol{\theta})\, d\boldsymbol{\theta} \\
E_2(z) &= \int_{p}^{\tau - f} \int_{\underline{\theta}_G}^{\frac{\theta_P + f - p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} + \int_{\tau - f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau - p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta} \\
F_2(z) &= \int_{\frac{\tau - p}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, d\boldsymbol{\theta}.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{\partial D_2(z)}{\partial f} &= -\int_{\underline{\theta}_P}^{p} \frac{f}{\pi} h\left(\frac{f}{\pi}, \theta_P\right) d\theta_P - \int_{\frac{f}{\pi}}^{\frac{\tau - p}{\pi}} \pi\theta_G h(\theta_G, \pi\theta_G + p - f)\, d\theta_G - \int_{\frac{\tau - p}{\pi}}^{\overline{\theta}_G} \pi\theta_G h(\theta_G, \tau - f)\, d\theta_G \\
\frac{\partial D_2(z)}{\partial p} &= -\int_{\underline{\theta}_P}^{\tau - f} \frac{\tau - p}{\pi} h\left(\frac{\tau - p}{\pi}, \theta_P\right) d\theta_P + \int_{\frac{f}{\pi}}^{\frac{\tau - p}{\pi}} \pi\theta_G h(\theta_G, \pi\theta_G + p - f)\, d\boldsymbol{\theta} + \int_{\underline{\theta}_P}^{\tau - f} \frac{\tau - p}{\pi} h\left(\frac{\tau - p}{\pi}, \theta_P\right) d\theta_P \\
&= \int_{\frac{f}{\pi}}^{\frac{\tau - p}{\pi}} \pi\theta_G h(\theta_G, \pi\theta_G + p - f)\, d\boldsymbol{\theta} \\
\frac{\partial D_2(z)}{\partial \tau} &= \int_{\underline{\theta}_P}^{\tau - f} \frac{\tau - p}{\pi} h\left(\frac{\tau - p}{\pi}, \theta_P\right) d\theta_P - \int_{\underline{\theta}_P}^{\tau - f} \frac{\tau - p}{\pi} h\left(\frac{\tau - p}{\pi}, \theta_P\right) d\theta_P + \int_{\frac{\tau - p}{\pi}}^{\overline{\theta}_G} \pi\theta_G h(\theta_G, \tau - f)\, d\theta_G \\
&= \int_{\frac{\tau - p}{\pi}}^{\overline{\theta}_G} \pi\theta_G h(\theta_G, \tau - f)\, d\theta_G,
\end{aligned}
$$

so,

$$
\begin{aligned}
\frac{\partial D_2(z^*)}{\partial f} &= -\int_{\underline{\theta}_P}^{p^*} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \pi^*\theta_G h(\theta_G, p^*)\, d\theta_G \\
\frac{\partial D_2(z^*)}{\partial p} &= 0 \\
\frac{\partial D_2(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \pi^*\theta_G h(\theta_G, p^*)\, d\theta_G,
\end{aligned}
$$

which is the same as in (B8).

Similarly,

$$\begin{aligned}
\frac{\partial E_2(z)}{\partial f} &= -\int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c)\, h(\theta_G, \tau - f)\, d\theta_G + \int_p^{\tau-f} (\theta_P - c)\, h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&\quad + \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c)\, h(\theta_G, \tau - f)\, d\theta_G \\
&= \int_p^{\tau-f} (\theta_P - c)\, h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial E_2(z)}{\partial p} &= -\int_{\underline{\theta}_G}^{\frac{f}{\pi}} (p - c)\, h(\theta_G, p)\, d\theta_G - \int_p^{\tau-f} (\theta_P - c)\, h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&\quad - \int_{\tau-f}^{\overline{\theta}_P} (\theta_P - c)\, h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial E_2(z)}{\partial \tau} &= \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c)\, h(\theta_G, \tau - f)\, d\theta_G - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c)\, h(\theta_G, \tau - f)\, d\theta_G \\
&\quad + \int_{\tau-f}^{\overline{\theta}_P} (\theta_P - c)\, h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&= \int_{\tau-f}^{\overline{\theta}_P} (\theta_P - c)\, h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial E_2(z^*)}{\partial f} &= 0 \\
\frac{\partial E_2(z^*)}{\partial p} &= -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c)\, h(\theta_G, p^*)\, d\theta_G - \int_{p^*}^{\overline{\theta}_P} (\theta_P - c)\, h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
\frac{\partial E_2(z^*)}{\partial \tau} &= \int_{p^*}^{\overline{\theta}_P} (\theta_P - c)\, h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P,
\end{aligned}$$

which is the same as the expressions in (B9).

Finally,

$$\begin{aligned}
\frac{\partial F_2(z)}{\partial f} &= \int_{\frac{\tau-p}{\pi^*}}^{\overline{\theta}_G} (\pi\theta_G + \tau - f - c)\, h(\theta_G, \tau - f)\, d\theta_G \\
\frac{\partial F_2(z)}{\partial p} &= \int_{\tau-f}^{\overline{\theta}_P} (\tau - p + \theta_P - c)\, h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial F_2(z)}{\partial \tau} &= -\int_{\tau-f}^{\overline{\theta}_P} (\tau - p + \theta_P - c)\, h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} (\pi\theta_G + \tau - f - c)\, h(\theta_G, \tau - f)\, d\theta_G,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial F_2(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (\pi^*\theta_G + p^* - c)\, h(\theta_G, p^*)\, d\theta_G \\
\frac{\partial F_2(z^*)}{\partial p} &= \int_{p^*}^{\overline{\theta}_P} (f^* + \theta_P - c)\, h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
\frac{\partial F_2(z^*)}{\partial \tau} &= -\int_{p^*}^{\overline{\theta}_P} (f^* + \theta_P - c)\, h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} [\pi^*\theta_G + p^* - c]\, h(\theta_G, p^*)\, d\theta_G
\end{aligned}$$

which is the same as the expressions in (B10). Since all the components are identical the result follows.