Diversification Benefits of REITs in Portfolio Allocation by REIT Property Types

Zicheng Ye, Diana Song[1]

Department of Economics, UNC at Chapel Hill

Abstract

This paper examines the existence of the diversification benefits of REIT common stocks by REIT asset property type using a utility-based framework in which investors are attached with varying risk aversions. The main focus is to address the question whether REIT property types matter when picking REIT stocks for generating additional portfolio wealth. We conduct our analysis using data from 1998 to 2016 to consider the long-term effect of REITs in diversifying portfolios, and analyze the performance of optimal mean-variance portfolios based on investor's risk aversion level and the access to different asset classes. The optimal weight and wealth compensation results show three main results. First, Self-Storage and Hotel stand out to be the two REIT property types that evidently improve the overall portfolio wealth when proper investment strategies are applied. Second, Self-Storage, Manufactured and Residential REITs become more wealth compensating with the increment of risk aversion level. Finally, bonds downplay the role of REITs when investors are of moderate to high risk aversion when given access to it. We conclude REITs of different property types offer varying benefits for portfolio construction, the extent to which depends on available investment tools, investment strategies and risk aversion level.

---

[1] Zicheng Ye, Department of Economics, UNC-Chapel Hill, zicheng@live.unc.edu; Diana Song, Department of Economics, UNC-Chapel Hill, songyw@live.unc.edu

1   Introduction

As the United States is still recovering from the housing bubble crisis at the end of last decade and speculators have predicted a new bubble coming in the nearest future, many investors expect that real estate sector will outperform in 2017. Investors usually have a proclivity for holding the stock market equivalents of hard assets due to the liquidity and convenience to manage. Based on such expectations, we are interested in Real Estate Investment Trusts (REITs), a market representative for the real estate sector. REITs provide high dividend and a potential for moderate, long-term capital appreciation. By conventional wisdom, they are less correlated with the traditional stock market since the two prices are driven by separate factors. As for the volatility, however, REITs can be no less volatile than traditional property investments, as any single REIT has different weaknesses in that specific type of real estate. For example, Retail REITs can see their net income plummet if a recession hits and several retailers collapse; Residential REITs can be volatile due to the uncertainty of immigration policies on the demand for rental housing; Hotel REITs has a higher seasonal demand that affects its stock return. Therefore, we think that it is crucial to understand how the financial market works for the REITs if we intend to use them for portfolio construction.

There are many unaddressed questions regarding the impact of REIT property type diversification. In our research, we intend to tentatively measure the evidence for the presence, or absence of REIT property-type diversification benefits, by focusing on the portfolio optimization of having different types of REIT properties. Our research will be a significant reference for investors who are looking for efficiently improving their portfolios by including

specific REIT property-typed stocks. The practical implication is that if there is only one REIT property type that improves the portfolio, then investors can save time and money (costs) investing into one instead of all REIT types. Such strategy will also depend on the risk exposure, as risk-loving investors love to gamble on risky assets while risk-averse investors think of hedging and safety as the priorities. Our research model therefore considers the level of risk as a factor to affect investor's choice of REIT stocks. We find Self-Storage stands out to be the dominant REIT property type that drives up the portfolio wealth under the long-only condition and Hotel REITs, when short selling constraints are removed, are favorable for risk loving investors as a hedging tool.

Section 2 reviews the relevant literatures on REIT portfolio diversification, followed by a discussion of the methodologies and data we utilize in Section 3. The empirical findings for each property type are analyzed in Section 4, which precedes the final conclusion and potential improvements.

2   Literature Review

Major literature precedents have touched upon the potential diversification benefits of having REITs in an equity portfolio to reduce the diversifiable risk and increase the overall portfolio return. For example, Kuhle (1987) examined the risk reduction in mixed-asset portfolios by distributing 26 Equity REITs, 16 Mortgage REITs and 42 common stocks to randomly created portfolios. He concluded that REIT portfolios are more efficient, in a Markowitz sense, than portfolios of only common stocks. However, the overall performance of the mixed portfolios consisting common stocks and REITs is not significantly different from that of portfolios

including common stocks only. Kuhle, Bhuyan, etc. (2014) further used mean-variance utility function analysis with various degrees of risk aversion to measure the impact of REITs in creating optimal portfolios. They found the change of weights in REITs in the optimal portfolio is more significant than the correlation between REITs and stock performance. Moreover, investors with risk aversion between 1 and 6 are better off investing almost entirely in REITs, short selling the bond and investing little in stock.

We found fewer literatures that show if such diversification benefit is attributed to specific REIT property types, as the annual REIT investment performance displays a disparate return pattern for each property sector. Rees and Kestel (2014) showed that there is a cointegration structure between and within Residential REITs and Retail REITs, and the cointegrated assets can be profitable under certain circumstances. Some literatures analyzed the role of the Diversified REIT in stock portfolio, a property type of REITs including less than 75% of a specific REIT type in its asset holding, but they have not mentioned if the same benefit applies to the individual REIT property types. Intuitively, if a portfolio contains 4 Office REITs, 4 Retail REITs, 4 Diversified REITs and 12 common stocks, is it possible that only those 4 Office REITs help generating the portfolio wealth?

We have found some closer literatures comparing the diversification benefit of Specialized REITs with that of Diversified REITs. Benefield, Anderson and Zumpano (2009) compared whether the stock market performance of REITs that have chosen to diversify by property type differs from the stock market performance of REITs that have become more specialized in their real estate holdings with the criteria of "Double" Sharpe Ratio, Jensen's Alpha and Treynor Index. They articulated the bias of the Sharpe Ratio in terms of addressing annual return and therefore used Double Sharpe Ratio as a replacement, which adjusts for small sample bias in estimating the standard Sharpe Ratio using annualized data. The results indicated a difference in performance between property-type diversified REITs and property-type specialized REITs, which depends on the overall market conditions varying by sample periods. Specifically, the diversified REITs perform better when overall markets are performing well. Furthermore, they noticed the property-type diversified REITs tended to have a high percentage of their portfolio to office property and less percentage in apartment and retail properties. Consequently, the uneven distribution of the property types in a portfolio could affect the overall performance during a

certain time period. We notice that a potential improvement of their research is the inclusion of common stocks and bonds. The measure for REIT-only portfolio performance is a valuable reference but with less real world practice since rational investors will not limit their portfolio choice to REITs stocks only. The same limitation exists in Katja's research (2015), in which he found the risk-adjusted portfolio return of the Specialized REITs is lower than that of the Diversified REITs. In 2016, Ping and Abdul reviewed the property type diversification strategy of Malaysian REITs. They assumed property type is one of the determinants for the performance of Malaysian REITs. However, this research was less relevant to the asset allocation strategy, and the Malaysian REITs are less mature, indicated by the number of their REIT samples. Another research by Ro and Alan (2011) examined the performance of specialized REITs and Diversified REITs from 1997 to 2006. They concluded there is no statistical difference between Specialized REITs and Diversified REITs in terms of the abnormal returns, even with the consideration of the leverage impact and the time span. Such finding is inconsistently with Benefield's. However, based on their CAPM and four-factor model, they did spot Specialized REITs have a significantly higher market risk than the Diversified REITs. We believe such difference in the market risk is a reason why different REIT property types can function distinctively in their portfolios under a utility-based framework.

Our research is more closely inspired and mainly based on a more recent study by Boudry, deRoos, and Ukhov (2015). They studied the diversification benefits of Preferred REIT stocks and Common REIT stocks using a utility-based framework, by setting up portfolios combining REIT preferred and common stocks with other equity and bond indices with reasonable constraints. There are three main results. First, REIT common stock allows investors with low risk aversion to form higher return portfolios while REIT preferred stock benefits investors with higher risk aversion. Second, REIT preferred stock provides a higher level of utility and replaces the role of REIT common stock and investment grade bond in a utility maximized portfolio when given access to it, which contradicted the conventional wisdom that preferred stock is merely a bond substitute. Finally, imposing constraints on portfolio settings would change the weights of asset allocations, which proved the existence of REIT industry. Based on their portfolio settings and empirical results, we therefore hypothesize that REIT stocks by property type could perform

distinctively under such utility-based framework with different risk-aversion level, and hence providing portfolio diversification benefits.

3 Methodology & Data

3.1 Methodology

We will follow the approach used by Boudry, deRoos, and Ukhov to analyze the effect of adding or removing one single REIT property type on portfolio wealth. The two criteria we will refer to are the maximized wealth level and the compensation ratio. The basic setup is identical to Boudry's model, in which we consider a simple constrained portfolio problem with two risky assets with expected returns $\mu 1$, $\mu 2$, standard deviations $\sigma 1$, $\sigma 2$, and their correlation ρ. We also consider investors with a conventional negative exponential utility of wealth $U(W) = -e^{-a_i w}$ as that in Boudry, where $a_i > 0$ is risk aversion level for investor i. We maximize the terminal wealth achieved by asset allocations, given an initial endowment of W0. By default, we set W0=1. An investor chooses investments in the two assets, $w1$ and $w2$, to maximize expected utility,

$$\underset{\substack{w_1, w_2 \\ w_1 \geq 0,\, w_2 \geq 0 \\ w_1 + w_2 = 1}}{\text{Max}} [(\tilde{W})] = \underset{\substack{w_1, w_2 \\ w_1 \geq 0,\, w_2 \geq 0 \\ w_1 + w_2 = 1}}{\text{Max}} -e^{-a_i(\mu_p - 0.5\, a_i \sigma_p{}^2)}$$

where μp and σp are the expected return and standard deviation of the portfolio with $w1$ invested in asset one and $w2$ invested in asset two. To solve this maximization problem, we solve

$$\underset{\substack{w_1, w_2 \\ w_1 \geq 0,\, w_2 \geq 0 \\ w_1 + w_2 = 1}}{\text{Max}} (\mu_p - 0.5\, a_i \sigma_p{}^2)$$

by forming the Lagrangian and differentiating it with respect to $w1$ and $w2$. The interior solutions to the problem are:

$$W_1 = \frac{\mu_1 - \mu_2 + a_i W_0 \left(\sigma_2{}^2 - \rho\sigma_1\sigma_2\right)}{a_i(\sigma_1{}^2 - 2\rho\sigma_1\sigma_2 + \sigma_2{}^2)}$$

$$W_2 = \frac{-\mu_1 + \mu_2 + a_i W_0 \left(\sigma_1{}^2 - \rho\sigma_1\sigma_2\right)}{a_i(\sigma_1{}^2 - 2\rho\sigma_1\sigma_2 + \sigma_2{}^2)}$$

The analytical solutions above show the optimal weight allocation counting toward the wealth maximization of a portfolio composed of two assets. In our model, we will have the investable universe including REIT property type focused indices, market equity indices, bond indices and riskless assets. We find the optimal weights for each asset in a portfolio for the sake of maximizing the expected utility. The derived solution for the optimal weights are different to the two-asset analytical solutions above, but the intuition behind solving for optimal weights for multiple assets will be analogous to the two-asset problem.

To assess the diversification benefits of different REIT assets to investors of different risk aversion, we employ Boudry's compensation ratios. The investor achieves derived utility of wealth,

$$(W_0) = E[U(\tilde{W})],$$

where n denotes that investor had access to the asset set with n assets. We restrict the investor from investing in one of the assets, namely the REIT property type whose diversification benefits are being assessed. The investment opportunity set now contains $(n-1)$ assets and the investor achieves derived utility of wealth under the constrained set,

$$V_{n-1}(W_0) = E[U(\tilde{W})].$$

Wealth compensation, ΔWk is the additional wealth required when asset k is removed from the investment opportunity set to restore utility to the level achieved under the initial asset set, and is the solution to the equation, $V_{n-1}(W_0 + \Delta W_k) = V_n(W_0)$. Analytically speaking, since the conventional utility function $U(W) = -e^{-a_i w}$ is solely based on the portfolio's wealth and the risk aversion level, as long as two portfolios have the same risk aversion, the wealth difference is just the wealth compensation that ensures two portfolios having the same utility level.

We use the wealth compensation because the intuition behind the wealth compensation is similar to the certainty equivalent, which is the guaranteed amount of cash that yield the same expected utility as a given risky asset with absolute certainty. Wealth compensation is the guaranteed amount of wealth that yield the same expected utility before removing one asset from the portfolio.

To evaluate the relative impact of different REIT assets in the investment opportunity set, we consider *compensation ratio* -- a ratio of compensation required for removal of asset k relative to compensation required for removal of asset j,

$$Compensation\ Ratio = \frac{\Delta W_k}{\Delta W_j}.$$

In our case, $\dfrac{\Delta W_{(losing\ one\ REIT\ property\ asset)/}}{\Delta W_{(losing\ all\ REIT\ assets)}}$

For example, we examine compensation ratios (as a function of risk aversion) of the wealth required to compensate investors for losing access to Hotel REITs by the compensation required to compensate investors for losing access to all REIT assets. In this sense the ratios measure the relative importance of Hotel REITs to an investor over different levels of risk aversion.

3.2   Portfolio Set up

We set up the optimization model using the standard Markowitz mean-variance framework where investors are able to freely borrow and lend. The typical portfolio performance measurement criterion under the mean-variance framework is the change of the Sharpe Ratio with the inclusion or exclusion of the access to certain assets in the predetermined investable universe. Again, since we are using utility and wealth level change for assessing portfolio's risk-return characteristics, which by themselves adjust portfolio risk and investor's risk aversion, we choose not to look at the Sharpe Ratio, which focuses on explaining the reasons for excess return. Figure 3.1 and 3.2 show the maximum wealth level obtained by the inclusion of each REIT property focused index in the base portfolio[2] composed of non-REIT indices (market equities, bonds and riskless assets), under different risk aversion level and under the precondition

---

[2]   We define the base portfolio as a portfolio that is composed of non-REIT equity indices, bond indices (depends on its accessibility) and riskless assets.

that short selling is allowed. For the non-REIT part of the portfolios (namely everything in the base portfolio), we are inspired by Boudry's constrained analysis and will therefore tell the portfolio maximization stories based on two conditions: Risk Free Asset and No Bonds, Risk Free Asset and Bonds. Such division addresses the real-world difficulty of accessing the corporate bonds but considers the inclusion of riskless assets for hedging purpose. Figure 3.1 shows the case with the access to bond and Figure 3.2 tells the case without the access to bond. Table 3 shows the wealth and portfolio weights obtained under different risk aversion level for our base portfolio containing no REITs and Table 4.1 and 4.2 shows the case for including all REIT property types[3]. We allow both long and short strategies, but we analyze them separately. In order to prevent the potentially high leverage, we set the weight constraints between -1 and 1 for each index used in portfolio construction. For the long-only case, we set weight between 0 and 1 to rule out the short position.

In Figure 3.1 and 3.2, the y-axis represents the maximum wealth level reached by the portfolio with the inclusion of one specific REIT property type. The x-axis shows the risk aversion level. The overall downwards trend is consistent with the modern portfolio theories: when risk aversion level increases, investor would allocate more resources in riskless assets or bonds and therefore the overall wealth level reduces as riskless assets and bonds generate relatively lower return.

To illustrate the benefits of the diversification we graph the compensation ratios in Figure 5.1 and 5.2. We include N-1 REIT property types in a portfolio with the same subdivisions[4] as set in the earlier portfolio maximization and measure the compensation ratios only when short selling is allowed[5]. The detailed observation will be articulated in section 4.2. Figure 5.1 visualizes the change of the compensation ratio by different REIT properties when people get access to bond. Figure 5.2 represents the change of the compensation ratio when bond becomes inaccessible. We would look at both the absolute level of the compensation ratio, which compares the wealth compensation for each REIT property type, and the change in the compensation ratio, which

---

[3] By saying including all REIT property types, or equivalently All-REIT portfolio, we mean a portfolio that is composed of non-REIT equity indices, bond indices (depends on its accessibility), riskless assets and all the REIT property focused indices. Basically, a portfolio that contains all investable assets we selected.

[4] Four subdivisions and their conditions: with bond+ long-only, with bond+ short, without bond+ long-only, without bond+ short-allowed

[5] When we conduct the optimization test for All-REIT portfolio, we find under the long-only case, all REIT property types, except for Self-Storage, have 0 weight. As a result, the wealth compensation for them will be the same.

compares the wealth compensation at different risk aversion level. The upward sloping curve tells such REIT property would become more preferred when investors are more risk averse.

3.3 Data

Under the same utility based framework and methodology, we follow Boudry's data selection and added some new variables of interest to our dataset. We collect daily, monthly and annual price value for 14 indices from January 1998 to December 2016. Datastream provides the MSCI World Ex-US index, the Russell 2000 growth index, the Russell 2000 value index, the Russell Mid Cap growth index, and the Russell Mid Cap value index. From SNL we collect the SNL Hotel REIT index, SNL Healthcare REIT index, SNL Industrial REIT index, SNL Retail REIT index, SNL Diversified REIT index, SNL Office REIT index, SNL Self-Storage REIT index, SNL Manufactured Homes REIT index, SNL Multifamily REIT index, SNL Residential REIT index and SNL Preferred REIT index. From WRDS/CRSP we obtain the monthly 30-day T-Bill. From Yahoo Finance, we collected the adjusted historical price for S&P 500, Russell 2000 and Russell Midcap index. We consider the varying market size of REITs and include Russell 2000 and Russell Midcap to present stocks with similar market capitalization size. From St. Louis Fed, we collected monthly BofA Merrill Lynch US High Yield Total Return Index and BofA Merrill Lynch US Corporate Master Total Return Index as substitutes for Barclays Indices used by Boudry. The essential differences between Barclays and BAML bond indices are the number of issues and reinvestment and they affect less on the return patterns.

Table 1 displays descriptive statistics for the indices used in our analysis. They include the crude proxy for the means, standard deviations, skewness, Kurtosis and percentiles for all REIT property indices over the past 5 years. One limitation of our descriptive data analysis is the method for finding the expected return. Since we draw the sample data from different resources with different time-span precision. We did not use the standard annualized expect return, but instead averaged the price change as a proxy for the average expected return. The intuition is that the general return pattern and hence the correlation would be similar. From that table, we can see both the cross-time and cross-sectional variation for the selected REIT property type indices over a selected sample period. Figure 1 visually presents the monthly historical returns of 4 randomly

selected REIT property type indices at a randomly selected time span, in which we can see some extreme return values for certain REIT property types. For example, in December 2008, Industrial REIT had a 55% increase in return from 47.46 to 73.59, after a previous 52.8% loss in October. In April 2009, Hotel REIT index embraced a 66.4% (10.4 to 17.3) return and was continually growing in the following years. On average, of those 11 REIT related indices, Self-Storage REITs have the highest average monthly return (1.04%), followed by Retail REITs (0.69%) and Multifamily REITs (0.66%), while Manufactured Homes and Diversified REITs tended to have a stable return, indicated by their relatively lower standard deviations (0.054 and 0.056). On the same time horizon, Russell related indices had an over 0.67% monthly return and S&P 500 on average had 0.46% return. The Bond indices unsurprisingly displayed a low mean-variance profile, with less than 0.5% return and 3% standard deviation. The risk-free rate approximated by the 30-day Treasury Bill was 0.16%. One noticeable observation is the risk-return pattern for Multifamily REITs and Residential REITs. Both indices showed a similar short-term and long-term return and variance pattern, despite the fact that there is no overlapping constituent of these two indices over the few years when we closely looked at their components. Similar results were obtained in the later correlation analysis. Given the similar mean-variance pattern and covariance structure, we assume Multifamily and Residential REITs would function analogously in portfolio optimization, thus dropping the Multifamily REIT index for the later optimization.

Table 2.1 and 2.2 reports the correlation among REIT property type indices. We believe the correlations between two REIT sectors are changing across time, same for that between REIT and the market. Table 2.1 displays four 5-year correlation matrices based on the average monthly return data because we believe there is going to be a cross-time variation in correlation. It is assisted with Figure 2.1 to 2.4 to graphically represent the change of correlation between any two REIT property type indices (we set Residential REITs, Hotel REITs, Self-Storage REITs and Manufactured Homes as four basis). Table 2.2 shows the average correlation between two indices. At first glance, Hotel, Manufactured Homes and Self-Storage had relatively lower, though still above 0.5 correlation with other properties. Preferred REIT[6], except for the case of

---

[6] Due to our research focus on specific REIT property types, we decide to drop SNL REIT Preferred REIT index.

Multifamily versus Residential, has almost always had the highest correlation with other property assets. Despite Hotel REITs and Diversified REITs had low correlation with the other REITs, their average correlations with the market return, indicated in Table 2.2, are evidently higher than those of other REITs. The difference in the correlation level with the market indices indicates the possibility of different REIT-Market stock combinations in a portfolio, which would have distinctive return-variance patterns.

The time varying nature of the correlation further indicates the usefulness of knowing the role of each REIT property type in a portfolio containing REITs. In 2015, the average monthly returns for Self-Storage REITs and Residential REITs were 2.73% and 1.08%, while those values fell dramatically to -0.78% and -0.05% in 2016. Hotel and Diversified REITs, on the other hand, surged from -2.41% and -1.14% in 2015 to 1.59% and 0.88% in 2016. The market return, approximated by Russell 2000 and S&P 500, increased from -0.41% and 0.01% to 1.62% and 0.8% in the same period. Clearly, we spot some REITs with return direction opposite to that of the market return and see potential diversification opportunities. The cross-sectional and cross-time variation of the REIT return further convince us that different REIT properties would have varying benefits to portfolio allocation.

4   Empirical Results and Analysis

4.1   Optimization Result

Inspired by the cross-time and cross-sectional difference in the return pattern and the disparate covariance structure, we construct portfolios and test their wealth level with the consideration of investor's risk aversion. Generally, REIT provides benefits whose extent depends on REIT property type. We observe there are some REIT property types that set themselves apart from others, indicated in the optimal weights and in the compensation ratio where some curves lay far beyond others, which echoes our initial hypothesis that REITs by their property types provide

varying benefits diversifying portfolios. The removal of short selling constraint would yield new investment scopes, namely the hedging opportunity. In the following subsections, we group properties based on their performance in portfolio and analyze each property type in detail.

4.2.1 Self-Storage REITs

We observe that for both the wealth optimization and compensation ratio, Self-Storage REITs stand out to the highest one across almost all risk aversion level with either the inclusion or the exclusion of bond indices. Toward the low risk aversion end in Figure 4.1 and 4.2, the 100% weight allocation on Self-Storage REITs and the 0% in non-REIT assets show that it could help the portfolio reach the maximum wealth at 0.0095 (0.0203)[7] if investors allocate all their wealth into Self-Storage REITs. The maximum wealth obtained for the base portfolio is 0.0067 (0.0162) that means the inclusion of Self-Storage REITs will lead to a 41% (27.4%) increase in investor's wealth level. Investors with higher risk aversion reduces the weight allocated to Self-Storage REITs, however, which still takes a fair amount of weight between 0.5 and 0.1. Similar portfolio allocation pattern can be observed when we remove the buy-only constraint. Initially investors would short sell riskless assets and bonds to have more weights on equity and REITs to obtain a higher expected return. The weight on Self-Storage REITs is 1 for the risk aversion level up until 1.5, at which level the weight of other equities drops to 0.2: this observation indicates a better return-variance structure of Self-Storage REITs in a portfolio than that of all other equities combined[8] and is consistent with Self-Storage's higher-than-market average monthly return. We do see an increasing rate of weight reduction and that explains why in Figure 3.1 the maximum wealth obtained by Self-Storage falls behind that obtained by other property types benefit from short selling. However, we do not see that in Figure 3.2, for which case bonds are excluded. In our long-only All-REIT portfolio, Self-Storage is again the only player in portfolio maximization. The terminal wealth as well as the asset allocation are identical to those of the portfolio that includes Self-Storage as the sole REIT asset. That also explains why we drop the

---

[7] The number inside a bracket indicates the case when bond becomes inaccessible

[8] The weights for the equity or bond indicated in the tables are the cumulative sum of the weights of each equity index or bond index, with each index having either a positive or non-positive (include 0) weight depends on the optimization simulation

compensation ratio test for long-only case as the ratio would always be 1 for Self-Storage REITs and 0 for other properties.

Based on the above evidence, we conclude Self-Storage REITs are outperforming, whose higher than market return, decent volatility and relatively low correlation with the market could dramatically drive up the portfolio wealth. However, when we compare Figure 3.1 and 3.2, we observe when people get access to bond for hedging against their increasing risk aversion, the riskier they are, the more likely they will divert funds away from Self-Storage to ensure the overall portfolio safety.

The compensation ratio measures the relative effect of Self-Storage REITs when losing access to it. The outputs assist our conclusion above. Self-Storage is placed at the top, meaning the wealth compensation for losing Self-Storage REITs in an All-REIT portfolio is the highest compared with that of losing any other REIT property type across almost all risk aversion levels. At risk level of 5, 34% of the wealth loss is explained by the loss of Self-Storage, while the second and the third, Hotel REITs and Diversified REITs occupy 34% when combined. That implies the wealth loss by dropping Self-Storage REITs out of the portfolio is equivalent to the wealth loss when investors throw away both Hotel and Diversified REITs. Moreover, the compensation ratio increases with the addition in risk level. The highest value lies above 37.3% (38.0%)[9]: the first implication is the inclusion of Self-Storage to generate return and investors would need to sacrifice the allocation of other properties to ensure portfolio safety; the second implication is derived from the earlier research by Boudry that indicated REIT Preferred Stocks tend to have an increasing compensation ratio pattern when risk aversion increases. We found the detailed composition in the MCSI Preferred REIT index, of which the top 10 assets in terms of size contain 5 Self-Storage REITs. Based on our compensation ratio test, we may say the benefits of the REIT Preferred Stocks could be partly explained by the performance of the Self-Storage assets.

4.2.2   Manufactured Homes

---

[9] a% (b%): "a" represents for the long-only case and "b" represents for the short selling case

We noticed Manufactured Homes act abnormally under the traditional buy-only situation. When risk aversion level is between 0.5 and 1, investors divert themselves from the bond market and Manufactured Homes REITs to the traditional equity market. That shows Manufactured Homes, in terms of its return benefits, is less comparable to other equities and Self-Storage REITs we analyzed above. However, start from risk aversion level 1.5, we surprisingly see in Figure 4.1 the 4.7% Manufactured Homes REITs in the portfolio. That number is even higher when bond is excluded (7.9%). The inclusion of Manufactured Homes REITs does not necessarily help obtain a higher wealth level that much, as when risk aversion level is 1.5, the terminal wealth is 0.005801 (0.0134)[10] compared with 0.005798 (0.00132) of our base portfolio: only 0.06% (0.8%) wealth improvement. Nevertheless, a constant inclusion of around 4% Manufactured Homes REITs when bond is included could potentially tell a preference for this REIT asset over bonds. People would be willing to include 4% of Manufactured Homes at extreme risk aversion level to sacrifice equities with higher return and bonds with less volatility because Manufactured Homes could have a better mean-variance structure, which becomes more important in a high-risky environment. When we remove the bond indices (Figure 4.2), the proportion of Manufactured Homes increases dramatically and could reach 24% at risk aversion 3.5, at which stage the weight on bond index is 0. It contributes at maximum a 6.2% (4.1%)[11] improvement in portfolio wealth. Based on such observation and finding, we say Manufactured Homes seems to be acting like an equity/bond hybrid.

When we look at the compensation ratio, Manufactured Homes REITs is among the lowest ones, especially at a low risk level. When investors can get access to bonds, the relative importance of Manufactured Homes increases when risk aversion level increases: that follows our 4% analysis, as Manufactured Homes takes the role of bonds and it becomes more important at a high-risk level. However, since its ability to generate portfolio return is lower than that of other properties such as Self-Storage and Hotel, its compensation ratios at absolute level (3% for long-only and 1% for short selling) are still low: if investors incorporate REITs of all property types in their portfolios (namely the All-REIT case), losing Manufactured Homes REITs would not harm the

---

[10] 0.0134 for the case when short selling is allowed. Same for 0.00132 and 0.8%
[11] 4.1% for short selling

portfolio significantly.

These results assist our analysis that Manufactured Homes REITs can partly take the role of bonds in a portfolio and acts as an equity/bond hybrid, though at a high risk level its hedging ability due to its nature as a REIT Common stock is lower than that of bonds and riskless assets.

4.2.3   Residential REITs and Retail REITs

Residential and Retail REITs are the remaining two types that investors with low to moderate risk aversion can benefit from under the long-only condition when investors can get access to all investment tools. Unlike Manufactured Homes REITs, the weights of Residential REITs and Retail REITs have a rapidly changing rate when risk aversion changes. As in Figure 4.1, for Retail REITs, its optimal weight dropped from 12.2% to 0% when risk aversion increased from 0.5 to 3.5. Residential REITs is by contrast more risk enduring, having a positive investing amount up until risk aversion reaches 8. When we alter the investable universe to exclude bond indices (Figure 4.2), Residential REITs and Retail REITs take a larger portion: Residential REITs index has its maximum weight 17.2% at risk aversion level of 3, while Retail REITs reaches 12.2% at the beginning and drops to 1.8% at an extreme risk aversion level. However, the benefits of including these two property types are not as significant as Self-Storage REITs. The maximized wealth obtained, both with bond and without bond, are apparently higher than that of our base portfolio, but the differences are not that dramatic: Residential REITs only provide at maximum 0.3% (2.7%)[12] wealth improvement and Retail REITs merely provide 0.4% (3.1%) improvement. Though those values increased to 2.9% (3.4%) for Retail and 1.4% (3.5%) for Residential when bond indices are removed and investors are of moderate risk aversion, they are tiny compared with those of Self-Storage and Manufactured Homes. The limited ability of Residential and Retail REITs to boost return would let them be quickly replaced by bonds and riskless assets to reduce the level of volatility in order to maximize wealth.

When we look at the compensation ratio, Retail REITs and Residential REITs stay between 0.04

---

[12] 2.7% for short selling case. Same for 3.1%, 3.4% and 3.5%

and 0.1. On average 7% of the wealth compensation for each property type is not insignificant but is somewhat of limited interest. Though the upward trend for Residential REIT's compensation ratio against risk aversion indicates its growing importance in an All-REIT portfolio, we view it as a potential Preferred-REIT feature, like what we observe in Manufactured Homes and Self-Storage. Therefore, we recommend investors to consider these two properties only if they want to include more than one REIT property type in their portfolio.

4.2.4   Industrial REITs and Healthcare REITs

Industrial and Healthcare are the two property types that based on the optimization results do not seem to generate much wealth for investors even with hedging possibility. Investors would draw all their money into the market equities and bonds when they are only allowed to long assets. The terminal wealth generated by these two REITs are merely slightly above that of our base portfolio. There is a leverage benefit though, indicated by the weight increase when short selling constraint is removed, but still the weights are too low to be considered significant, even when bonds are removed: either 17%[13] Healthcare and 32% Industrial failed to rival with 60% Retail and 53% Manufactured Homes, not to mention 100% Self-Storage. Their compensation ratio graphs further convince us that when investors have all REIT property types to choose for portfolio construction, losing Industrial or Healthcare REITs would on average only affect less than 3% wealth loss for the portfolio. Both the addition of either one property to a non-REIT portfolio, indicated by the terminal wealth improvement, and the removal of either one property from an All-REIT portfolio, indicated by the wealth compensation ratio, show the relatively low importance level of Industrial and Healthcare REITs in diversifying the portfolio.

4.2.5   Hotel REITs, Diversified REITs and Office REITs

Under the long-only constraint, the positive amount of REIT asset allocation in the other sectors does not indicate Hotel, Diversified and Office are of no use for portfolio wealth maximization.

---

[13] The highest weight value for Healthcare REITs across all risk aversion level

Rather, it shows when investors in reality cannot short and cover effectively (exp. short-squeeze), the other REIT property types could compete with the traditional equity market in terms of maximizing portfolio wealth as they have competitive return-risk frontiers and relatively lower correlations with the market. However, when investors can short-sell a reasonable amount of the REIT assets, Hotel, Diversified and Office REITs set themselves apart from almost all other property types. As one useful purpose for short selling is hedging, or protection, it mitigates the potential loss of a portfolio and we consider short selling as a normal investment strategy in our research.

Contrary to a better Diversified REITs performance over Specialized REITs performance stated in the mainstream literatures, under the utility-based framework and with the inclusion of a wider investable universe, we praise Hotel REITs over Diversified REITs for driving up investor's wealth. When investors love risk and invest in bonds, Hotel REITs (0.0191, 17.9%[14]) results in a higher wealth level, followed by Diversified REITs (0.0181, 11.8%) and Office REITs (0.0175, 8.1%). The negative wealth-risk slopes in Figure 5.1 and 5.2 for the three indices are analogous, and such ranking retains when risk aversion level is high. What worth mentioning in Figure 3.1 is the terminal wealth level obtained when investors get access to bonds. When the risk aversion level increases, the maximum wealth generated by the inclusion of Self-Storage REITs is outraced by that of Hotel, Diversified and Office REITs, whose short-selling strategy potentially count more hedging benefits than Self-Storage's long-only diversification strategy does. Despite more risk loving investors will still want to invest 100% in Self-Storage, when investors are of moderate risk aversion level, the wealth curve of the Hotel REITs lays above that of the Self-Storage REITs. When investors are more afraid of risk, they will also place Diversified and Office atop Self-Storage. Such observations show that with some degrees of risk aversion, when Hotel REITs is the sole REIT property in a portfolio, its ability to lift the overall wealth is dominant (maximum at 23.9% wealth improvement compare with 13.6% for Self-Storage at risk aversion level 4.5). It could be explained by Hotel REIT's relatively higher correlation with the equity indices and its high monthly volatility indicated by its standard deviation making it a perfect asset for short selling. The overall downward sloping compensation ratio curve for Hotel REITs indicates the common stock nature of Hotel REITs based on Boudry's analysis on the

---

[14] In regard to this presentation form, 0.0191 is the absolute level of wealth and 17.9% is the percentage improvement compare to the base portfolio.

compensation ratio for common stocks. Therefore, investors would need to regard Hotel REITs as a normal risky asset.

When investors have enough funds to invest into all REIT assets, the wealth compensation for losing Hotel REITs, though showing a downward trend, is still generally higher than that of Diversified and Office REITs. The benefits of short selling Hotel REITs in an All-REIT portfolio (Table 4.1) can be further magnified when we look at the optimal weights of asset allocation at a given risk aversion level. At the risk aversion level of 2.5, Hotel takes 91% short telling while Diversified and Office REITs each takes 100% short selling. The compensation ratio for Hotel REITs at risk aversion level of 2.5, however, is 23.9%, much higher than 17.8% for Diversified REITs and 16.1% for Office REITs. These information combined imply that losing Hotel REITs, that is not even 100% invested, needs a higher amount of compensation than losing either Diversified or Office REITs that is 100% fully invested. Between the risk aversion level of 1.5 and 2, we see in Figure 5.2 the compensation for losing Hotel REITs (around 30.9%) even exceeds that for losing Self-Storage (around 29.8%) when bond is excluded. That means at this risk aversion range, investors would rather giving up the best performing Self-Storage REITs than giving up Hotel REITs.

5   Conclusion and Improvements

We study the diversification benefits of having REIT stocks of different property types using a utility-based framework, which allows us to adjust for investor's segmentation due to the variant risk aversion. We allow both long and short position to access the asset allocation and diversification benefits of REIT properties in every investment possibility, thereby measuring the maximized wealth and optimal weights under different risk aversion levels. We analyze the performance for each REIT property type in two ways: the addition of a certain REIT property type to a non-REIT portfolio and the removal of a certain REIT property type from an All REIT portfolio. For the former case, we refer to the portfolio weights and maximum wealth level; for the latter case, we visualize and interpret the compensation required for losing access to that specific REIT property type.

We utilize data from January 1998 to December 2016. Previous research on REIT portfolios tend to ignore the varying size of REITs and only used S&P 500 as the market benchmark, and we fix it by including a variety of asset classes as represented by their index equivalents, including mid cap indices and small cap indices as we take the size of real estate investment trusts into consideration.

First, we find Self-Storage REITs stand out to be the property type that provides a significant amount of diversification benefits to a portfolio that contains Self-Storage REITs as the sole REIT property no matter what risk aversion level investors have. If investors can only choose REITs of one property type to include in their long-term portfolio, Self-Storage REITs is the one that enables investors to reach a higher wealth with the adjustment of risk aversion. Moreover, under the long-only condition, Self-Storage is the only property type investors will want to invest in even if they can choose assets from other REIT types. Our explanation for such phenomenon is the strict dominance of the Self-Storage REITs over other types of REITs except Manufactured Homes and Residential in terms of the expected return, standard deviation, and its lowest correlation with the market performance.

Second, we observe under the short selling conditions, Hotel REITs provides a hedging benefit and can outperform Self-Storage REITs and Diversified REITs when investors can access all investment tools, like bonds and riskless assets. Losing Hotel REITs in a portfolio will be even more compensating when investors love risk. However, Hotel REIT in a portfolio functions like a normal stock with high volatility and market correlation, therefore its relative importance is negatively related with the risk aversion level.

Third, the slope of the compensation ratios for each REIT property type shows that the benefit of having Preferred REITs at a high-risk aversion level drawn by precedent literatures can be attributed to Preferred REIT's underlying property types. Within our selected REIT universe, Manufactured Homes, Residential and Self-Storage REITs are among the top constituents of Vanguard REIT Index, S&P US Preferred REIT Stock Index and MSCI Preferred REIT Index, whose upward sloping compensation ratio curves are analogous to those of Manufactured Homes, Residential and Self-Storage REITs. The nature of the Preferred REIT stock and whether it is a hybrid security mixing up debts and equities is not addressed in this research paper as we

primarily focus on REIT property types. However, our observations for Manufactured Homes REITs and Residential REITs lean toward the concept of the equity-bond hybridity.

Finally, we do not find obvious portfolio diversification benefits of Healthcare and Industrial REITs given their relatively mediocre return-variance level. If investors have limited funds to distribute in REITs, Healthcare and Industrial REITs become the least favorable for long term investment.

Our research can function as an empirical reference for investors who want to use REIT stocks as an investment tool to diversify or hedge their portfolio. We attempt to simulate the portfolio construction as it is in real-world scenarios by allowing unconstrained investment strategies and investable financial instruments of all kind except derivatives. There are several shortcomings and unpreventable facts regarding the data and approach we use for measuring portfolio optimization. First of all, we consider the cross-time variability of each REIT industry by measuring the long-term returns, but each REIT property type may have its idiosyncratic benefits in any given year and therefore in the short period of time the simulated results can be completely different. The return-variance pattern can be dramatically different under different time periods. From 1998 to 2006 the monthly average of Hotel REITs is 0.0018, but that value more than doubled if we include 1995 to 1998 as Hotel REITs had its heyday at that period. The increase in average expected return combined with an even higher volatility can alter the results we obtain for the short selling portfolio allocation and drive up the compensation ratio for losing Hotel REITs. Therefore, a detailed analysis in the time variation by separating investor's horizon will be helpful explaining the performance of each property type in a portfolio.

Moreover, we use average monthly return by computing the average monthly price change as the proxy for expected return. Since we draw stock and bond data from various resources, the recorded number of days are not identical for each index. Therefore, we use monthly end price for computing expected monthly return. The intuition is that we compare the property types on a relative level, not an absolute level (we are not measuring what amount of REITs to include in a portfolio), so the general trend and portfolio allocation will be similar. Though such computation method is legitimate to tell the relative difference for the return and covariance structure of each REIT property type, it is better to annualize the return using daily data to prevent some

distortions caused by price fluctuation.

Since our intention is to prove the existence of the diversification benefits of REITs by property types, another issue is to define the property types. Major data vendors that provide REIT property focus index classify property type in different ways, either in names or in underlying companies. NAREIT for example, named Hotel REITs as Lodging REITs and has three other property types we have not included in our research: Data Center REITs, Infrastructure REITs and Specialty REITs[15]. For the consistency of our research, we follow Boudry's REIT data selection and pick SNL as the vendor.

An extension of this research is coming to analyze the potential factors. For example, dividend structure and seasonality that lead to such discrepancy in the property type diversification benefits. We will specifically focus on the cash flow variability by REIT sector and how it can potentially explain the discrepant REIT diversification benefits.

---

[15] NAREIT defines Specialty REITs as Equity REITs that own and manage a unique mix of property types and collect rent from tenants. Specialty REITs own properties that don't fit within the major REIT sectors.

References

Kuhle James. 1987. Portfolio Diversification and Return Benefits--Common Stock vs. Real Estate Investment Trusts (REITs), *The Journal of Real Estate Research* 2, 1-9.

Bhuyan Rafiqul, Kuhle James, Ikromov Nuriddin, Chiemeke Charles. 2014. Optimal Portfolio Allocation among REITs, Stocks, and Long-Term Bonds: An Empirical Analysis of US Financial Markets. *Journal of Mathematical Finance* 04. 104-112.

Rees Patrick, Kestel Sevtap. 2014. Analysis of portfolio diversification between REIT assets. Journal of Computational and Applied Mathematics 259, 425-433.

Benefield Justin, Anderson Randy, Zumpano Leonard. 2009. Performance differences in property-type diversified versus specialized real estate investment trusts (REITs). Review of Financial Economics 18. 70-79.

Ping Tiong, Abdul Rohaya. 2016. Property Types Diversification Strategy of Malaysian Real Estate Investment Trust. MATEC Web Conference 66. 00076.

Katja Oilkarinen. 2015. Does property-type diversification in REITs provide superior risk-adjusted returns if compared to specialized REITs? Evidence from the U.S. during different market conditions. Aalto Univeristy School of Business Department of Finance. Thesis 14205.

Ro, SeungHan & Ziobrowski, Alan. 2011. Does Focus Really Matter? Specialized vs. Diversified REITs. The Journal of Real Estate Finance and Economics. 42. 68-83.

Boudry Walter, deRoos Jan, Ukhov Andrey. 2015. Diversification Benefits of REIT Preferred and Common Stock: New Evidence from a Utility based Framework. 2015 NAREIT/AREUEA Real Estate Research Conference

Table 1

Descriptive Statistics for all REIT indices from 2012 to 2016

MH	2016	2015	2014	2013	2012	STOR	2016	2015	2014	2013	2012
Mean	0.010	0.018	0.030	0.007	0.004	Mean	-0.008	0.027	0.021	0.006	0.012
STD	0.051	0.049	0.052	0.058	0.044	STD	0.064	0.053	0.045	0.056	0.041
Median	0.020	0.016	0.022	0.018	-0.003	Median	-0.008	0.038	0.007	0.014	0.026
Skewness	-0.117	0.305	0.753	-0.596	0.086	Skew	0.425	0.022	0.886	-0.345	-0.586
Kurtosis	-1.056	-0.452	2.867	-0.478	-1.602	Kurtosis	-0.336	-0.956	0.773	-0.721	-0.173
Q99	0.091	0.104	0.144	0.083	0.065	Q99	0.108	0.112	0.115	0.085	0.072
Q95	0.072	0.094	0.115	0.074	0.060	Q95	0.091	0.104	0.094	0.075	0.057
Q75	0.048	0.045	0.037	0.049	0.047	Q75	0.032	0.057	0.049	0.041	0.040
Q25	-0.026	-0.015	0.010	-0.025	-0.031	Q25	-0.054	-0.019	-0.006	-0.029	-0.015
Q5	-0.061	-0.049	-0.030	-0.085	-0.051	Q5	-0.086	-0.047	-0.028	-0.082	-0.051
Q1	-0.068	-0.050	-0.061	-0.100	-0.060	Q1	-0.103	-0.049	-0.043	-0.088	-0.066

RES	2016	2015	2014	2013	2012	RET	2016	2015	2014	2013	2012
Mean	0.000	0.011	0.024	-0.005	0.003	Mean	-0.001	0.002	0.016	0.001	0.018
STD	0.047	0.039	0.043	0.034	0.039	STD	0.054	0.058	0.032	0.053	0.036
Median	-0.012	0.018	0.019	0.001	0.006	Median	-0.002	0.011	0.017	0.014	0.017
Skew	0.463	-0.177	-0.481	-1.234	0.432	Skew	0.338	0.160	-0.208	0.008	-0.361
Kurtosis	-0.765	-1.303	3.481	1.512	-0.765	Kurtosis	0.005	-1.462	1.429	-0.270	-0.441
Q99	0.083	0.064	0.104	0.037	0.074	Q99	0.094	0.087	0.073	0.093	0.066
Q95	0.073	0.064	0.082	0.031	0.057	Q95	0.084	0.082	0.064	0.077	0.064
Q75	0.029	0.034	0.040	0.018	0.032	Q75	0.017	0.036	0.028	0.023	0.046
Q25	-0.033	-0.027	0.013	-0.013	-0.034	Q25	-0.030	-0.050	-0.002	-0.037	0.001
Q5	-0.060	-0.042	-0.029	-0.065	-0.039	Q5	-0.072	-0.067	-0.026	-0.077	-0.038
Q1	-0.063	-0.046	-0.067	-0.080	-0.040	Q1	-0.090	-0.073	-0.046	-0.083	-0.047

OFF	2016	2015	2014	2013	2012	IND	2016	2015	2014	2013	2012
Mean	0.007	-0.001	0.017	0.003	0.009	Mean	0.018	0.001	0.014	0.005	0.022
STD	0.058	0.049	0.039	0.037	0.038	STD	0.057	0.056	0.047	0.051	0.051
Median	0.013	-0.004	0.011	0.014	0.001	Median	0.012	-0.002	0.009	0.009	0.034
Skew	0.058	-0.197	-0.349	-0.726	0.154	Skew	0.345	0.328	0.499	-0.032	-0.582
Kurtosis	-0.313	-0.495	2.894	-0.856	0.518	Kurtosis	-0.430	-0.943	1.994	-1.421	0.422

Q99	0.107	0.076	0.090	0.048	0.077	Q99	0.120	0.092	0.111	0.087	0.098
Q95	0.091	0.066	0.072	0.041	0.069	Q95	0.101	0.089	0.091	0.076	0.081
Q75	0.038	0.035	0.032	0.030	0.027	Q75	0.055	0.035	0.024	0.053	0.055
Q25	-0.031	-0.024	0.005	-0.021	-0.009	Q25	-0.021	-0.047	-0.005	-0.042	-0.011
Q5	-0.077	-0.073	-0.033	-0.056	-0.041	Q5	-0.057	-0.071	-0.043	-0.071	-0.050
Q1	-0.087	-0.086	-0.065	-0.058	-0.058	Q1	-0.066	-0.072	-0.069	-0.079	-0.079

HT	2016	2015	2014	2013	2012	HC	2016	2015	2014	2013	2012
Mean	0.016	-0.024	0.021	0.018	0.009	Mean	0.002	-0.012	0.021	-0.008	0.012
STD	0.078	0.037	0.043	0.041	0.059	STD	0.061	0.059	0.051	0.061	0.037
Median	0.021	-0.017	0.023	0.028	0.026	Median	-0.008	-0.019	0.009	-0.007	0.016
Skew	-0.217	0.513	0.048	-0.468	-0.368	Skew	0.090	0.250	0.140	-0.337	-0.022
Kurtosis	-1.335	1.020	0.820	-1.292	-0.644	Kurtosis	-0.737	-0.885	0.972	-1.222	-0.881
Q99	0.124	0.050	0.103	0.069	0.101	Q99	0.095	0.087	0.115	0.074	0.070
Q95	0.112	0.025	0.081	0.061	0.082	Q95	0.092	0.072	0.097	0.062	0.065
Q75	0.076	-0.009	0.043	0.053	0.042	Q75	0.046	0.035	0.049	0.045	0.034
Q25	-0.036	-0.048	0.005	-0.016	-0.034	Q25	-0.030	-0.063	-0.003	-0.041	-0.020
Q5	-0.099	-0.074	-0.041	-0.045	-0.080	Q5	-0.083	-0.084	-0.042	-0.100	-0.038
Q1	-0.100	-0.076	-0.055	-0.049	-0.083	Q1	-0.096	-0.098	-0.072	-0.104	-0.045

DIV	2016	2015	2014	2013	2012
Mean	0.009	-0.011	0.010	0.001	0.016
STD	0.050	0.041	0.034	0.038	0.030
Median	0.008	-0.008	0.011	0.010	0.012
Skew	0.691	-0.180	-0.330	-0.584	-0.130
Kurtosis	1.043	0.239	0.778	-1.244	0.443
Q99	0.114	0.059	0.068	0.039	0.064
Q95	0.086	0.042	0.057	0.039	0.062
Q75	0.028	0.012	0.028	0.035	0.031
Q25	-0.018	-0.035	-0.004	-0.031	0.001
Q5	-0.058	-0.071	-0.042	-0.056	-0.026
Q1	-0.060	-0.085	-0.056	-0.062	-0.041

Table 2.1

5-Year Average Monthly Correlation

2012-16	STOR	MH	MF	RES	RET	OFF	DIV	IND	HT	HC	PF	RUS	SP
STOR	1.00	0.71	0.79	0.78	0.72	0.63	0.63	0.64	0.33	0.68	0.79	0.16	0.27
MH	0.71	1.00	0.82	0.84	0.74	0.69	0.68	0.69	0.39	0.82	0.82	0.15	0.24
MF	0.79	0.82	1.00	1.00	0.69	0.74	0.69	0.67	0.41	0.75	0.82	0.23	0.25
RES	0.78	0.84	1.00	1.00	0.71	0.75	0.72	0.69	0.42	0.77	0.84	0.25	0.27
RET	0.72	0.74	0.69	0.71	1.00	0.83	0.79	0.83	0.53	0.80	0.94	0.28	0.43
OFF	0.63	0.69	0.74	0.75	0.83	1.00	0.90	0.88	0.69	0.76	0.92	0.56	0.59
DIV	0.63	0.68	0.69	0.72	0.79	0.90	1.00	0.88	0.67	0.75	0.91	0.57	0.63
IND	0.64	0.69	0.67	0.69	0.83	0.88	0.88	1.00	0.65	0.69	0.89	0.56	0.61
HT	0.33	0.39	0.41	0.42	0.53	0.69	0.67	0.65	1.00	0.36	0.62	0.73	0.67
HC	0.68	0.82	0.75	0.77	0.80	0.76	0.75	0.69	0.36	1.00	0.88	0.10	0.18
PF	0.79	0.82	0.82	0.84	0.94	0.92	0.91	0.89	0.62	0.88	1.00	0.39	0.50
RUS	0.16	0.15	0.23	0.25	0.28	0.56	0.57	0.56	0.73	0.10	0.39	1.00	0.83
SP	0.27	0.24	0.25	0.27	0.43	0.59	0.63	0.61	0.67	0.18	0.50	0.83	1.00

2007-11	STOR	MH	MF	RES	RET	OFF	DIV	IND	HT	HC	PF	RUS	SP
STOR	1.00	0.75	0.80	0.80	0.86	0.84	0.82	0.75	0.74	0.84	0.89	0.70	0.65
MH	0.75	1.00	0.75	0.77	0.77	0.76	0.81	0.77	0.68	0.81	0.82	0.76	0.73
MF	0.80	0.75	1.00	1.00	0.85	0.85	0.89	0.70	0.79	0.73	0.90	0.80	0.76
RES	0.80	0.77	1.00	1.00	0.86	0.86	0.89	0.71	0.80	0.74	0.91	0.80	0.77
RET	0.86	0.77	0.85	0.86	1.00	0.95	0.91	0.88	0.89	0.85	0.98	0.81	0.76
OFF	0.84	0.76	0.85	0.86	0.95	1.00	0.93	0.87	0.86	0.84	0.97	0.83	0.79
DIV	0.82	0.81	0.89	0.89	0.91	0.93	1.00	0.82	0.88	0.82	0.96	0.88	0.84
IND	0.75	0.77	0.70	0.71	0.88	0.87	0.82	1.00	0.77	0.85	0.90	0.78	0.74
HT	0.74	0.68	0.79	0.80	0.89	0.86	0.88	0.77	1.00	0.67	0.89	0.85	0.82
HC	0.84	0.81	0.73	0.74	0.85	0.84	0.82	0.85	0.67	1.00	0.88	0.69	0.67
PF	0.89	0.82	0.90	0.91	0.98	0.97	0.96	0.90	0.89	0.88	1.00	0.86	0.82
RUS	0.70	0.76	0.80	0.80	0.81	0.83	0.88	0.78	0.85	0.69	0.86	1.00	0.95
SP	0.65	0.73	0.76	0.77	0.76	0.79	0.84	0.74	0.82	0.67	0.82	0.95	1.00

2002-06	STOR	MH	MF	RES	RET	OFF	DIV	IND	HT	HC	PF	RUS	SP
STOR	1.00	0.58	0.57	0.59	0.72	0.66	0.74	0.66	0.59	0.69	0.75	0.33	0.15
MH	0.58	1.00	0.57	0.61	0.62	0.63	0.71	0.57	0.62	0.56	0.68	0.46	0.22
MF	0.57	0.57	1.00	1.00	0.76	0.82	0.77	0.78	0.66	0.61	0.87	0.57	0.46
RES	0.59	0.61	1.00	1.00	0.77	0.83	0.79	0.78	0.67	0.63	0.89	0.57	0.45
RET	0.72	0.62	0.76	0.77	1.00	0.86	0.88	0.90	0.61	0.80	0.95	0.46	0.26
OFF	0.66	0.63	0.82	0.83	0.86	1.00	0.86	0.87	0.69	0.73	0.95	0.56	0.42
DIV	0.74	0.71	0.77	0.79	0.88	0.86	1.00	0.82	0.75	0.81	0.94	0.65	0.47
IND	0.66	0.57	0.78	0.78	0.90	0.87	0.82	1.00	0.62	0.72	0.92	0.38	0.25
HT	0.59	0.62	0.66	0.67	0.61	0.69	0.75	0.62	1.00	0.66	0.75	0.65	0.47
HC	0.69	0.56	0.61	0.63	0.80	0.73	0.81	0.72	0.66	1.00	0.82	0.51	0.40
PF	0.75	0.68	0.87	0.89	0.95	0.95	0.94	0.92	0.75	0.82	1.00	0.58	0.41
RUS	0.33	0.46	0.57	0.57	0.46	0.56	0.65	0.38	0.65	0.51	0.58	1.00	0.82
SP	0.15	0.22	0.46	0.45	0.26	0.42	0.47	0.25	0.47	0.40	0.41	0.82	1.00

1998-01	STOR	MH	MF	RES	RET	OFF	DIV	IND	HT	HC	PF	RUS	SP
STOR	1.00	0.64	0.73	0.74	0.76	0.71	0.69	0.71	0.42	0.72	0.81	0.37	0.13
MH	0.64	1.00	0.55	0.60	0.62	0.58	0.39	0.50	0.21	0.56	0.59	0.20	0.10
MF	0.73	0.55	1.00	1.00	0.71	0.90	0.56	0.88	0.61	0.51	0.92	0.22	0.14
RES	0.74	0.60	1.00	1.00	0.73	0.91	0.56	0.88	0.60	0.53	0.92	0.23	0.14
RET	0.76	0.62	0.71	0.73	1.00	0.73	0.68	0.72	0.72	0.74	0.89	0.26	0.12
OFF	0.71	0.58	0.90	0.91	0.73	1.00	0.62	0.88	0.58	0.50	0.93	0.33	0.22
DIV	0.69	0.39	0.56	0.56	0.68	0.62	1.00	0.67	0.49	0.59	0.74	0.52	0.31
IND	0.71	0.50	0.88	0.88	0.72	0.88	0.67	1.00	0.60	0.54	0.90	0.35	0.31
HT	0.42	0.21	0.61	0.60	0.72	0.58	0.49	0.60	1.00	0.42	0.74	0.39	0.28
HC	0.72	0.56	0.51	0.53	0.74	0.50	0.59	0.54	0.42	1.00	0.67	0.14	0.03
PF	0.81	0.59	0.92	0.92	0.89	0.93	0.74	0.90	0.74	0.67	1.00	0.37	0.22
RUS	0.37	0.20	0.22	0.23	0.26	0.33	0.52	0.35	0.39	0.14	0.37	1.00	0.68
SP	0.13	0.10	0.14	0.14	0.12	0.22	0.31	0.31	0.28	0.03	0.22	0.68	1.00

Table 2.2

Average Monthly Correlations from 1998 to 2016

CORR	STOR	MH	MF	RES	RET	OFF	DIV	IND	HT	HC	PF	RUS	SP	IG	HY
STOR	1.00	0.69	0.74	0.75	0.79	0.75	0.75	0.69	0.61	0.76	0.83	0.46	0.38	0.28	0.33
MH	0.69	1.00	0.70	0.73	0.71	0.69	0.70	0.69	0.55	0.73	0.76	0.48	0.43	0.34	0.41
MF	0.74	0.70	1.00	1.00	0.81	0.84	0.80	0.71	0.71	0.68	0.89	0.55	0.51	0.24	0.42
RES	0.75	0.73	1.00	1.00	0.82	0.84	0.81	0.72	0.71	0.69	0.90	0.56	0.52	0.24	0.43
RET	0.79	0.71	0.81	0.82	1.00	0.91	0.85	0.87	0.79	0.80	0.96	0.56	0.51	0.36	0.54
OFF	0.75	0.69	0.84	0.84	0.91	1.00	0.87	0.86	0.79	0.76	0.96	0.63	0.58	0.36	0.57
DIV	0.75	0.70	0.80	0.81	0.85	0.87	1.00	0.78	0.77	0.77	0.91	0.70	0.62	0.35	0.57
IND	0.69	0.69	0.71	0.72	0.87	0.86	0.78	1.00	0.71	0.75	0.89	0.58	0.55	0.39	0.56
HT	0.61	0.55	0.71	0.71	0.79	0.79	0.77	0.71	1.00	0.58	0.83	0.67	0.61	0.26	0.61
HC	0.76	0.73	0.68	0.69	0.80	0.76	0.77	0.75	0.58	1.00	0.83	0.43	0.39	0.38	0.44
PF	0.83	0.76	0.89	0.90	0.96	0.96	0.91	0.89	0.83	0.83	1.00	0.63	0.58	0.36	0.57
RUS	0.46	0.48	0.55	0.56	0.56	0.63	0.70	0.58	0.67	0.43	0.63	1.00	0.83	0.16	0.62
SP	0.38	0.43	0.51	0.52	0.51	0.58	0.62	0.55	0.61	0.39	0.58	0.83	1.00	0.19	0.62
IG	0.28	0.34	0.24	0.24	0.36	0.36	0.35	0.39	0.26	0.38	0.36	0.16	0.19	1.00	0.49
HY	0.33	0.41	0.42	0.43	0.54	0.57	0.57	0.56	0.61	0.44	0.57	0.62	0.62	0.49	1.00

Note that STOR represents Self-Storage, MH represents Manufactured Home, MF for Multifamily, RES for Residential, RET for Retail, OFF for Office, DIV for Diversified, IND for Industrial, HT for Hotel, HC for Healthcare, PF for Preferred, RUS for Russell 2000, SP for S&P 500, IG for BAML Corporate Master Bond, HY for BAML High Yield Bond.

Table 3.1

Weight of each index for wealth maximization with/without the access to bond, base portfolio

Risk	Equity	Bond	Riskless	Wealth	Risk	Equity	Riskless	Wealth
0.5	1.48	0.52	-1.00	0.0162	0.5	2.00	-1.00	0.0159
1	0.93	1.07	-1.00	0.0145	1	2.00	-1.00	0.0133
1.5	0.69	1.31	-1.00	0.0133	1.5	1.98	-0.98	0.0111
2	0.56	1.44	-1.00	0.0123	2	1.47	-0.47	0.0095
2.5	0.48	1.52	-1.00	0.0115	2.5	1.15	-0.15	0.0084
3	0.43	1.57	-1.00	0.0108	3	0.94	0.06	0.0077
3.5	0.39	1.61	-1.00	0.0101	3.5	0.79	0.21	0.0071
4	0.36	1.56	-0.92	0.0095	4	0.70	0.30	0.0066
4.5	0.30	1.48	-0.78	0.0090	4.5	0.61	0.39	0.0061
5	0.26	1.41	-0.67	0.0086	5	0.54	0.46	0.0058
5.5	0.23	1.35	-0.58	0.0082	5.5	0.48	0.52	0.0054
6	0.21	1.30	-0.51	0.0078	6	0.44	0.56	0.0051
6.5	0.20	1.25	-0.45	0.0074	6.5	0.40	0.60	0.0049
7	0.19	1.20	-0.40	0.0072	7	0.38	0.62	0.0047
7.5	0.19	1.16	-0.35	0.0069	7.5	0.35	0.65	0.0045
8	0.19	1.13	-0.31	0.0067	8	0.33	0.67	0.0043
8.5	0.18	1.10	-0.28	0.0064	8.5	0.31	0.69	0.0042
9	0.18	1.07	-0.25	0.0063	9	0.30	0.70	0.0040
10	0.18	1.02	-0.20	0.0059	10	0.27	0.73	0.0038
11	0.17	0.99	-0.16	0.0056	11	0.24	0.76	0.0036
12	0.17	0.95	-0.12	0.0054	12	0.22	0.78	0.0035
15	0.16	0.88	-0.05	0.0047	15	0.18	0.82	0.0031
20	0.13	0.69	0.18	0.0040	20	0.14	0.86	0.0028

Note that Equity includes Russell 2000, Growth, Value, Russell Mid Cap, Growth, Value, S&P 500, MSCI ex-US Index. Bond includes Corporate Master Bond Index and High Yield Index. The weight value displayed above is the cumulative sum of the weights of each index.

Table 4.1

**Optimal Weights for Wealth Maximization with the Access to Bond, All-REIT Portfolio**

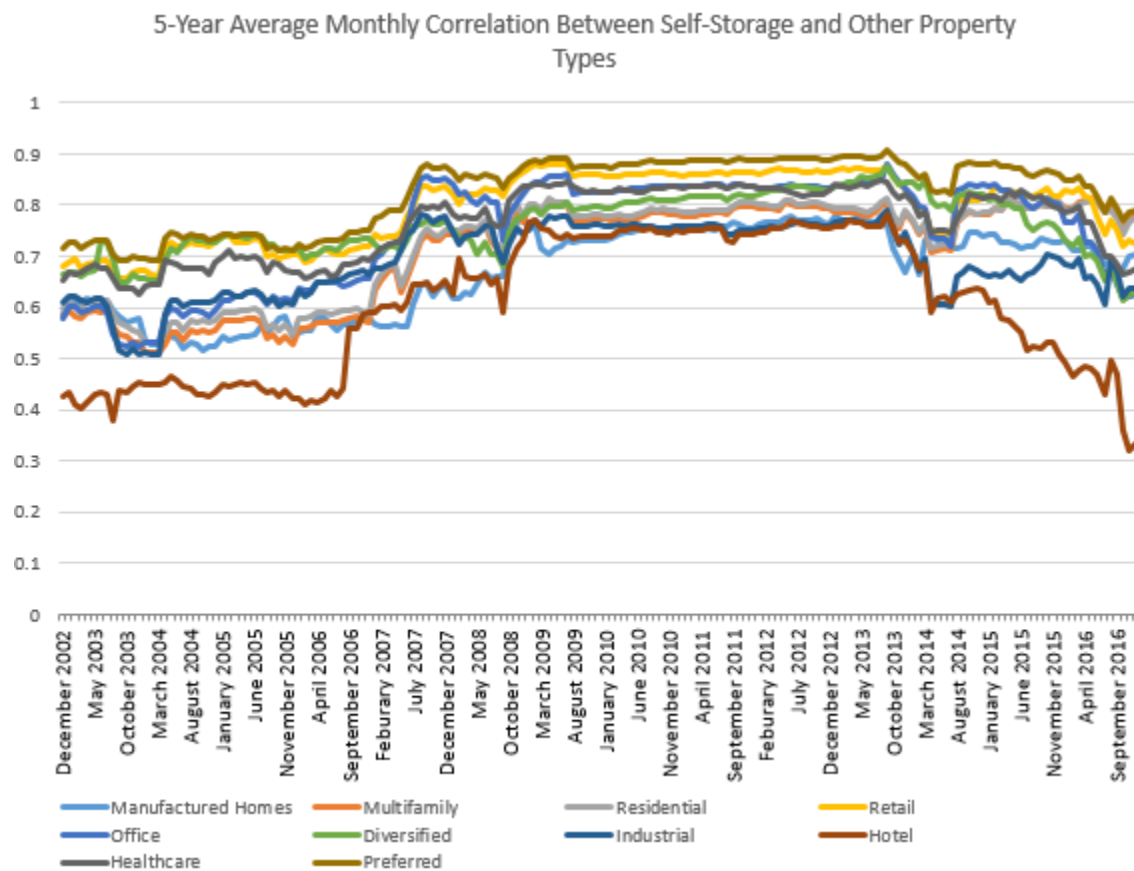| HC	HT	IND	DIV	RET	OFF	RES	STOR	MH	EQT	BOND	RF	Wealth
-1.00	-1.00	0.86	-1.00	1.00	-1.00	1.00	1.00	0.24	1.61	0.29	-1.00	0.0318
-0.69	-1.00	0.51	-1.00	1.00	-1.00	1.00	1.00	-0.13	1.42	0.88	-1.00	0.0292
-0.52	-1.00	0.39	-1.00	1.00	-1.00	0.96	1.00	-0.27	1.31	1.13	-1.00	0.0273
-0.43	-1.00	0.35	-1.00	1.00	-1.00	0.84	1.00	-0.29	1.29	1.24	-1.00	0.0256
-0.37	-0.91	0.29	-1.00	1.00	-1.00	0.72	1.00	-0.30	1.24	1.33	-1.00	0.0241
-0.34	-0.81	0.23	-1.00	1.00	-1.00	0.62	1.00	-0.29	1.19	1.40	-1.00	0.0228
-0.29	-0.73	0.22	-1.00	0.91	-1.00	0.58	1.00	-0.28	1.14	1.45	-1.00	0.0216
-0.25	-0.65	0.22	-1.00	0.80	-1.00	0.56	1.00	-0.26	1.09	1.48	-1.00	0.0205
-0.21	-0.58	0.22	-1.00	0.71	-1.00	0.55	1.00	-0.26	1.04	1.52	-1.00	0.0194
-0.19	-0.53	0.22	-1.00	0.65	-1.00	0.55	1.00	-0.25	1.00	1.55	-1.00	0.0185
-0.17	-0.48	0.22	-1.00	0.59	-1.00	0.54	1.00	-0.25	0.97	1.58	-1.00	0.0176
-0.16	-0.45	0.21	-1.00	0.53	-0.94	0.52	1.00	-0.24	0.93	1.60	-1.00	0.0167
-0.15	-0.43	0.19	-0.92	0.48	-0.89	0.50	0.95	-0.23	0.88	1.62	-1.00	0.0159
-0.14	-0.41	0.17	-0.86	0.43	-0.84	0.48	0.90	-0.22	0.84	1.64	-1.00	0.0151
-0.13	-0.39	0.16	-0.80	0.40	-0.80	0.46	0.86	-0.20	0.81	1.64	-1.00	0.0144
-0.12	-0.38	0.14	-0.76	0.37	-0.75	0.44	0.82	-0.18	0.78	1.64	-1.00	0.0138
-0.12	-0.36	0.12	-0.72	0.34	-0.72	0.43	0.78	-0.17	0.76	1.64	-1.00	0.0132
-0.11	-0.35	0.11	-0.68	0.32	-0.68	0.41	0.75	-0.16	0.74	1.61	-0.96	0.0126
-0.10	-0.31	0.10	-0.62	0.29	-0.62	0.38	0.67	-0.15	0.68	1.51	-0.84	0.0117
-0.09	-0.28	0.09	-0.57	0.26	-0.57	0.35	0.61	-0.14	0.64	1.43	-0.74	0.0109
-0.08	-0.25	0.08	-0.53	0.24	-0.53	0.32	0.56	-0.13	0.60	1.37	-0.65	0.0103
-0.07	-0.20	0.06	-0.43	0.19	-0.44	0.27	0.45	-0.11	0.51	1.23	-0.47	0.0087
-0.05	-0.14	0.05	-0.34	0.14	-0.34	0.21	0.34	-0.09	0.42	1.06	-0.26	0.0071

Note that EQT represents Equity and RF represents Riskless Assets.

Table 4.2

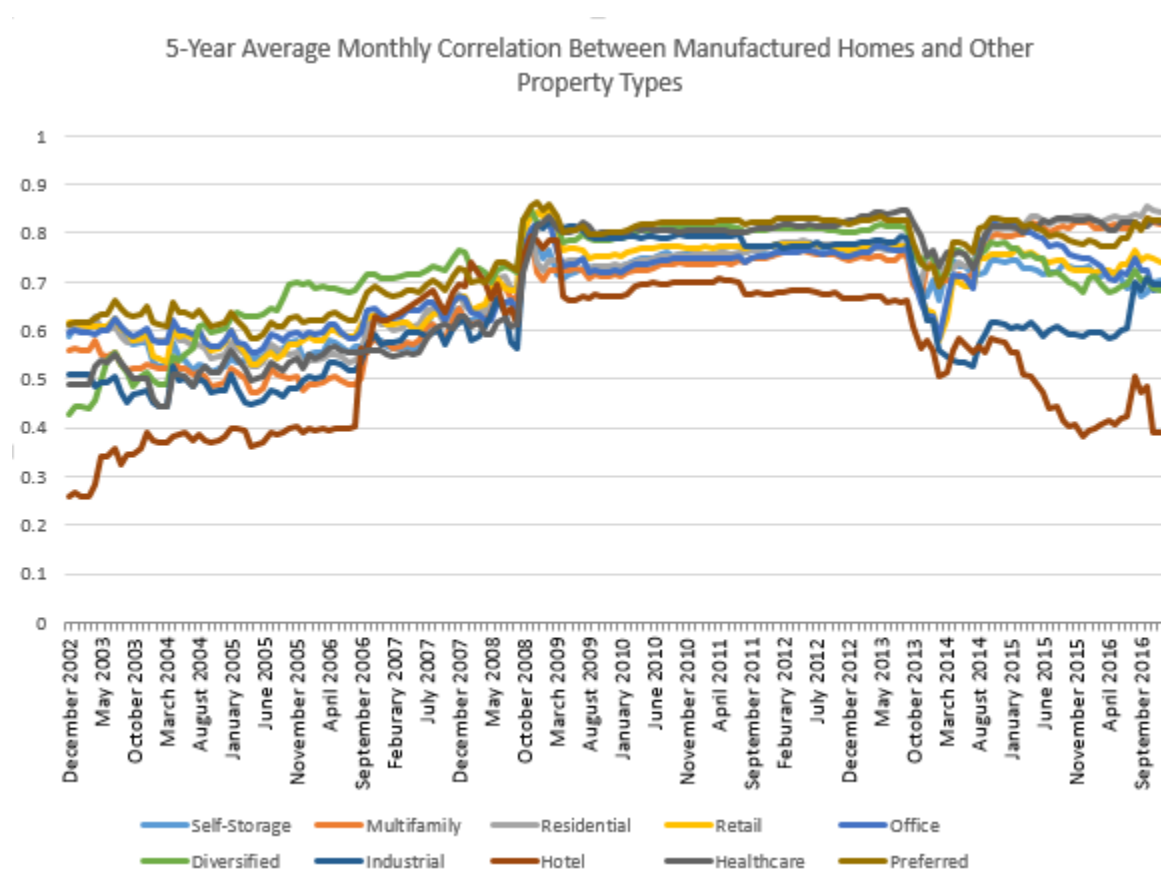**Optimal Weights for Wealth Maximization without the Access to Bond, All-REIT Portfolio**

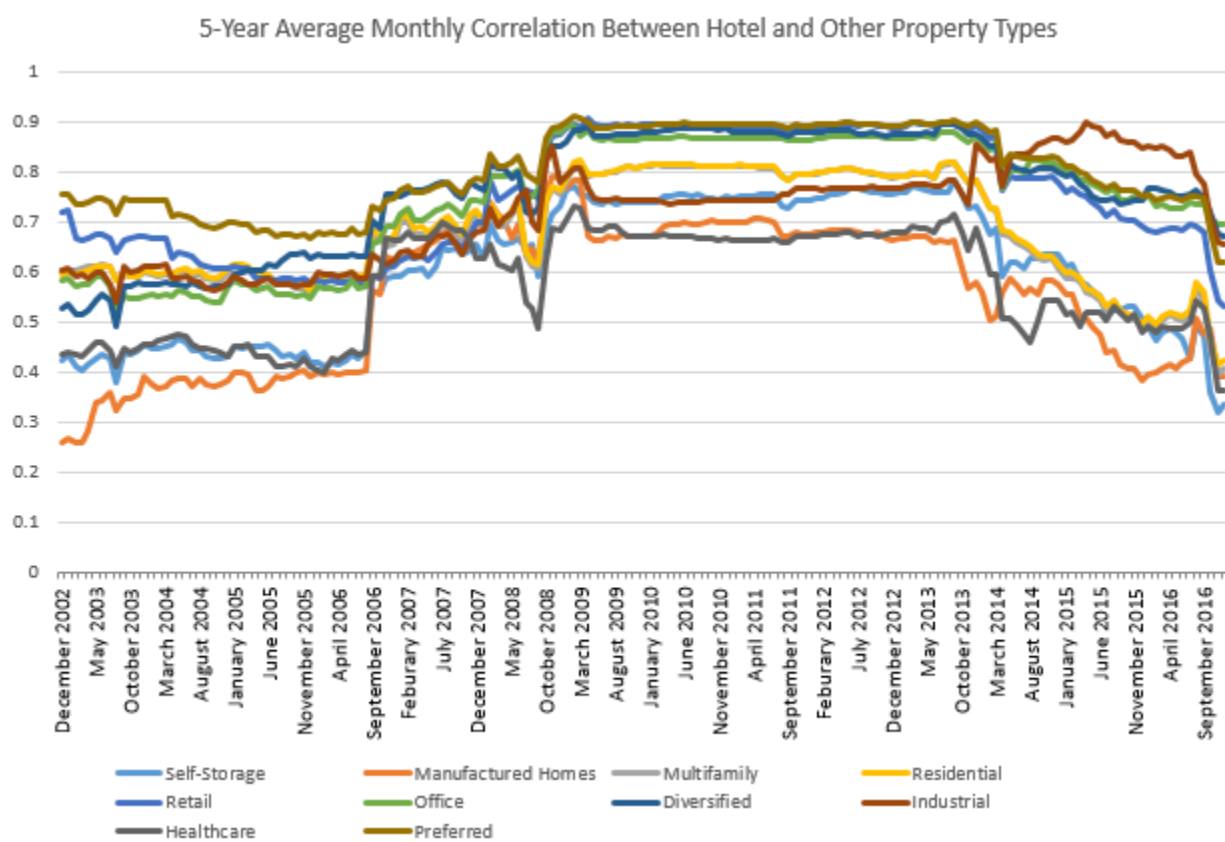| HC	HT	IND	DIV	RET	OFF	RES	STOR	MH	EQT	RF	Wealth
-1.00	-1.00	0.87	-1.00	1.00	-1.00	1.00	1.00	0.33	1.80	-1.00	0.0317
-0.56	-1.00	0.43	-1.00	1.00	-1.00	1.00	1.00	0.05	2.07	-1.00	0.0288
-0.35	-1.00	0.30	-1.00	1.00	-1.00	0.86	1.00	-0.01	2.20	-1.00	0.0266
-0.24	-1.00	0.25	-1.00	1.00	-1.00	0.74	1.00	-0.02	2.27	-1.00	0.0245
-0.17	-1.00	0.22	-1.00	1.00	-1.00	0.66	1.00	-0.02	2.31	-1.00	0.0225
-0.13	-0.99	0.19	-1.00	1.00	-1.00	0.60	1.00	-0.02	2.30	-0.97	0.0206
-0.12	-0.86	0.18	-1.00	0.97	-1.00	0.52	1.00	-0.05	2.06	-0.72	0.0189
-0.09	-0.74	0.20	-1.00	0.85	-1.00	0.49	1.00	-0.06	1.88	-0.53	0.0174
-0.07	-0.64	0.22	-1.00	0.76	-1.00	0.46	1.00	-0.07	1.73	-0.38	0.0161
-0.06	-0.57	0.23	-1.00	0.69	-1.00	0.44	1.00	-0.08	1.62	-0.27	0.0150
-0.05	-0.51	0.24	-0.97	0.62	-0.99	0.41	1.00	-0.09	1.51	-0.17	0.0139
-0.06	-0.47	0.23	-0.89	0.54	-0.91	0.36	0.99	-0.10	1.39	-0.08	0.0129
-0.06	-0.43	0.22	-0.82	0.50	-0.84	0.33	0.92	-0.09	1.28	0.00	0.0121
-0.06	-0.40	0.20	-0.76	0.46	-0.78	0.31	0.85	-0.08	1.19	0.07	0.0113
-0.05	-0.37	0.19	-0.71	0.43	-0.73	0.29	0.79	-0.08	1.11	0.13	0.0107
-0.05	-0.35	0.17	-0.67	0.40	-0.69	0.27	0.75	-0.07	1.05	0.18	0.0102
-0.05	-0.33	0.16	-0.63	0.38	-0.65	0.26	0.70	-0.07	0.99	0.23	0.0097
-0.04	-0.31	0.16	-0.59	0.36	-0.62	0.24	0.66	-0.06	0.93	0.27	0.0092
-0.04	-0.28	0.14	-0.53	0.32	-0.56	0.22	0.60	-0.06	0.84	0.34	0.0085
-0.04	-0.25	0.13	-0.48	0.29	-0.51	0.20	0.54	-0.05	0.76	0.40	0.0079
-0.03	-0.23	0.12	-0.44	0.27	-0.47	0.18	0.50	-0.04	0.70	0.45	0.0074
-0.03	-0.18	0.09	-0.35	0.21	-0.38	0.15	0.40	-0.03	0.56	0.56	0.0063
-0.02	-0.13	0.07	-0.26	0.16	-0.29	0.11	0.30	-0.02	0.43	0.66	0.0051

Figure 1

Performance of Selected REIT Property Focused Index from 2007 to 2014
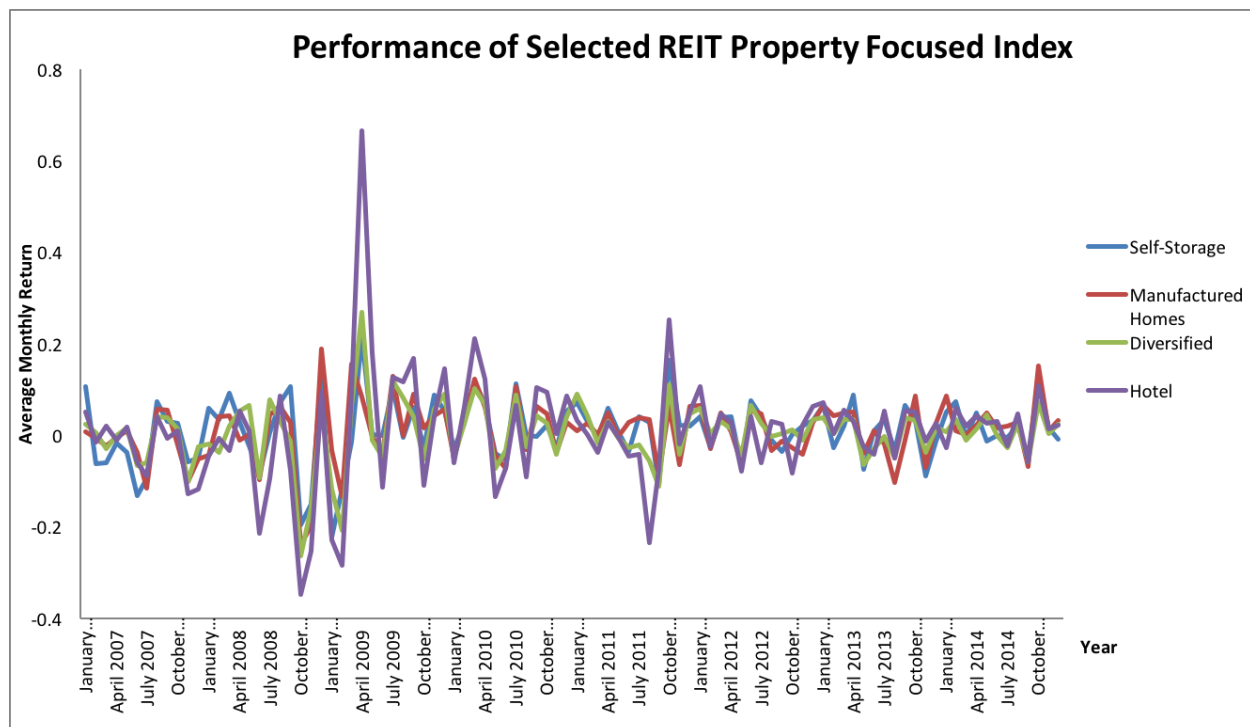
Figure 2

5-Year Average Monthly Correlation between One REIT Sector and Other REIT Sectors
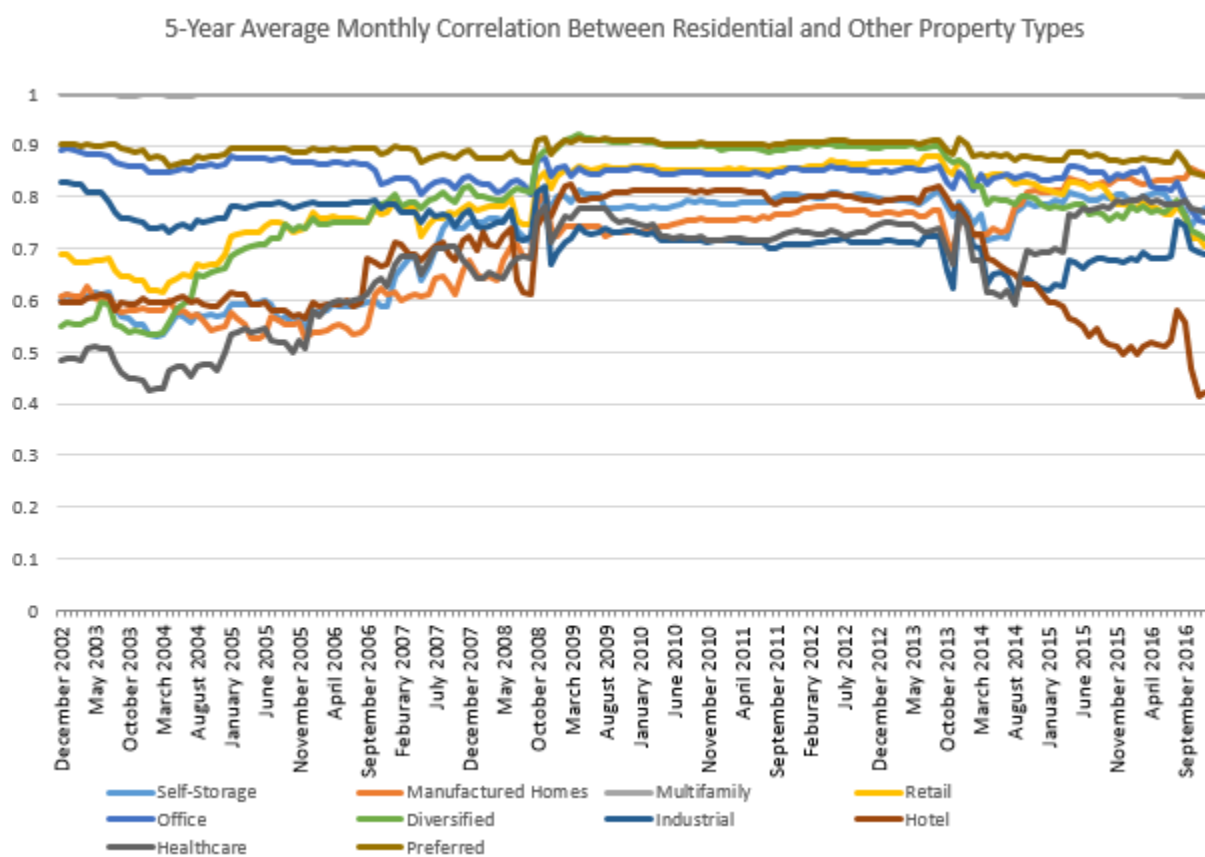
2.1 Self-Storage

5-Year Average Monthly Correlation Between Self-Storage and Other Property Types

2.2 Manufactured Homes

5-Year Average Monthly Correlation Between Manufactured Homes and Other Property Types

2.3 Hotel

5-Year Average Monthly Correlation Between Hotel and Other Property Types

Self-Storage   Manufactured Homes   Multifamily   Residential
Retail   Office   Diversified   Industrial
Healthcare   Preferred

2.4 Residential

5-Year Average Monthly Correlation Between Residential and Other Property Types

Figure 3.1

Maximized Portfolio Wealth Obtained by Including One REIT Property Type, with Bond

Porfolio Wealth Obtained By Different REIT Property Types With the Acess to Bond
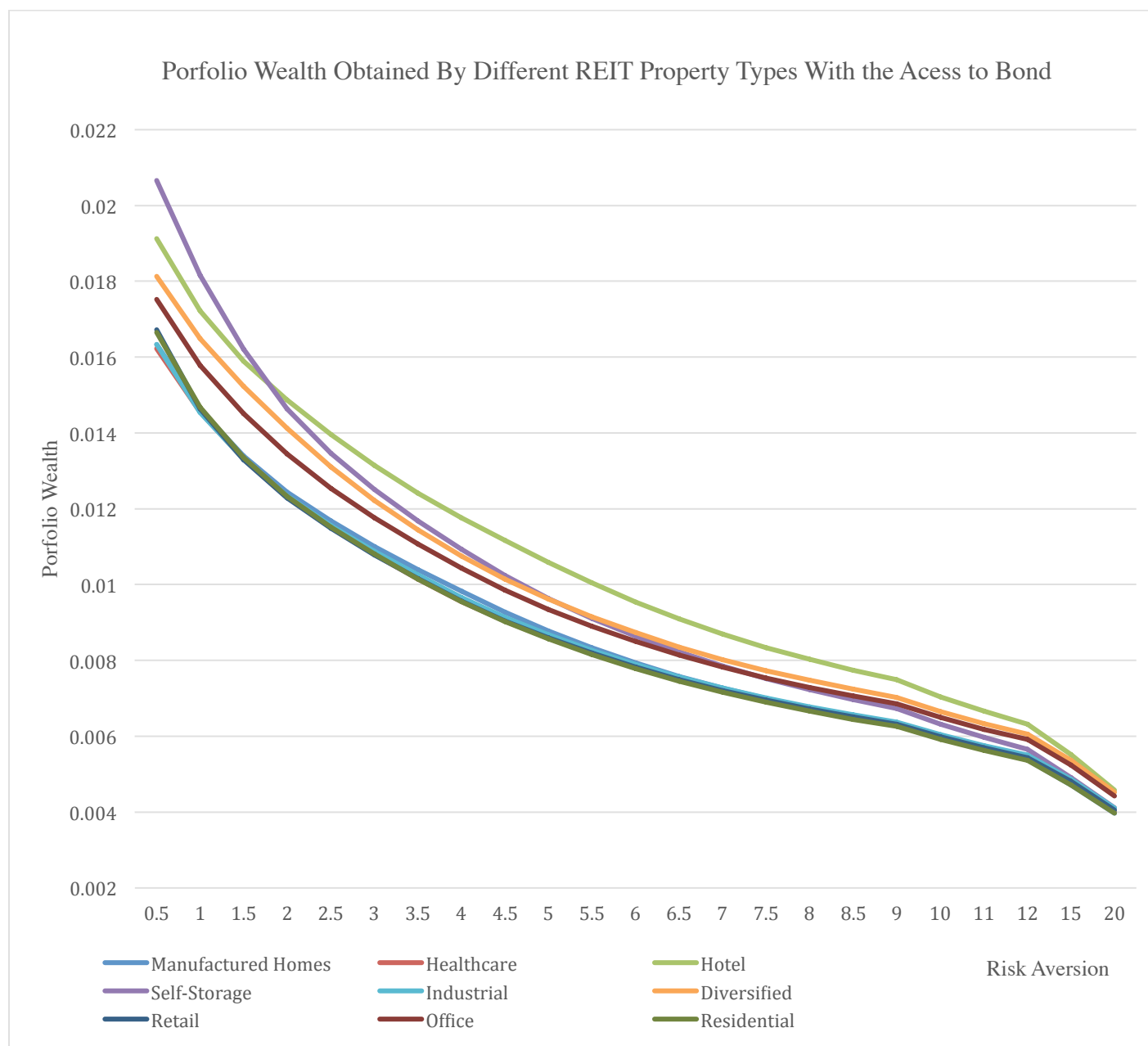
Figure 3.2

Maximized Portfolio Wealth Obtained by Including One REIT Property Type, without Bond

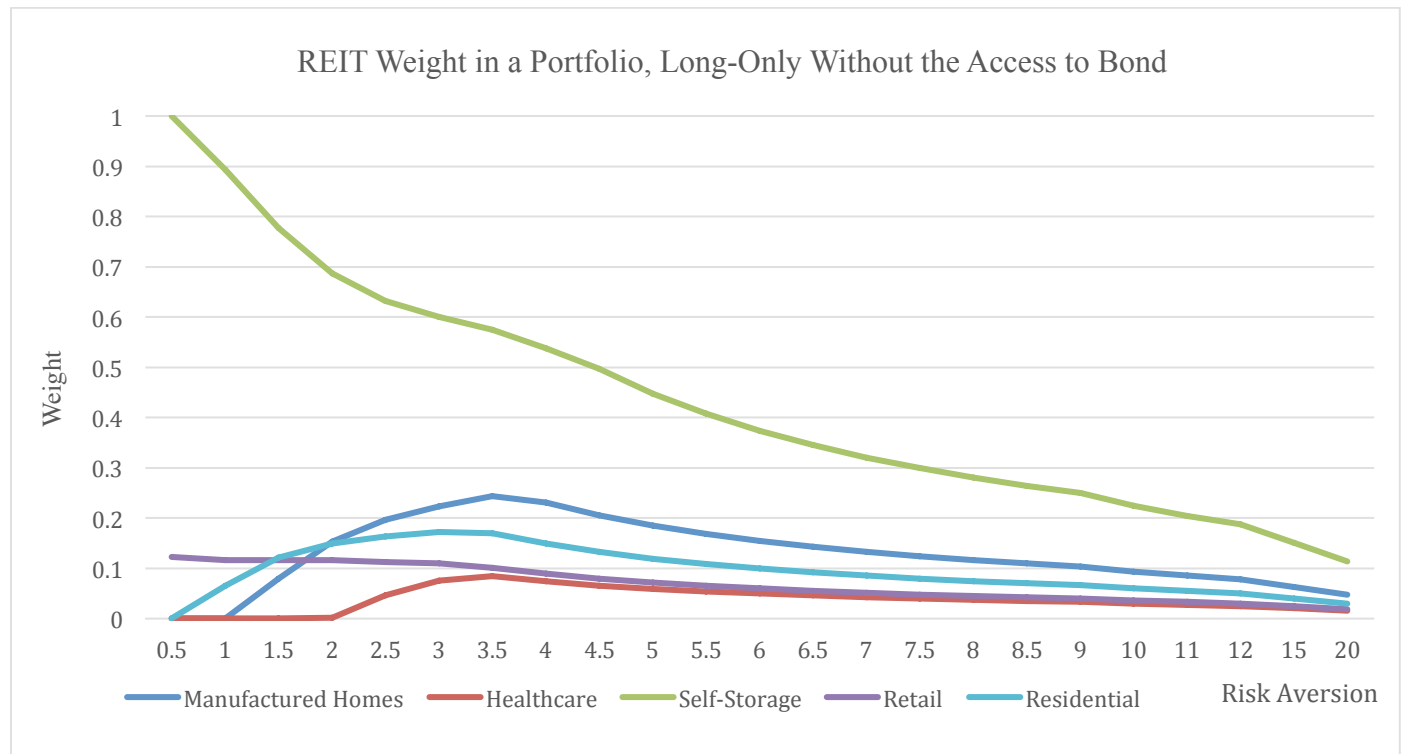Portfolio Wealth Obtained By Different REIT Property Types Without the Access to Bond
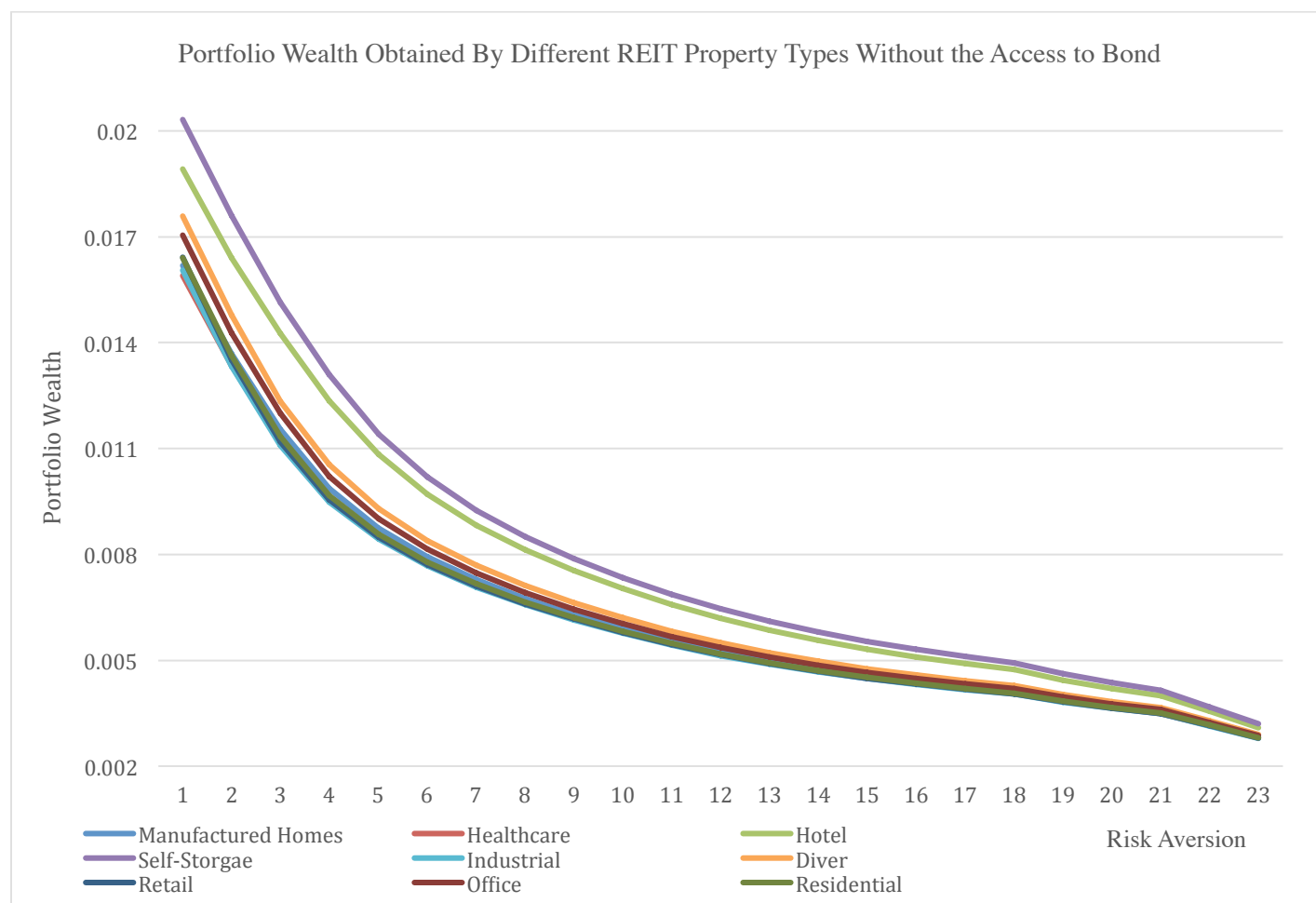
Figure 4.1

**Optimal Weights of REITs in Long Only Portfolio Optimization with Bond,
By Property Type**

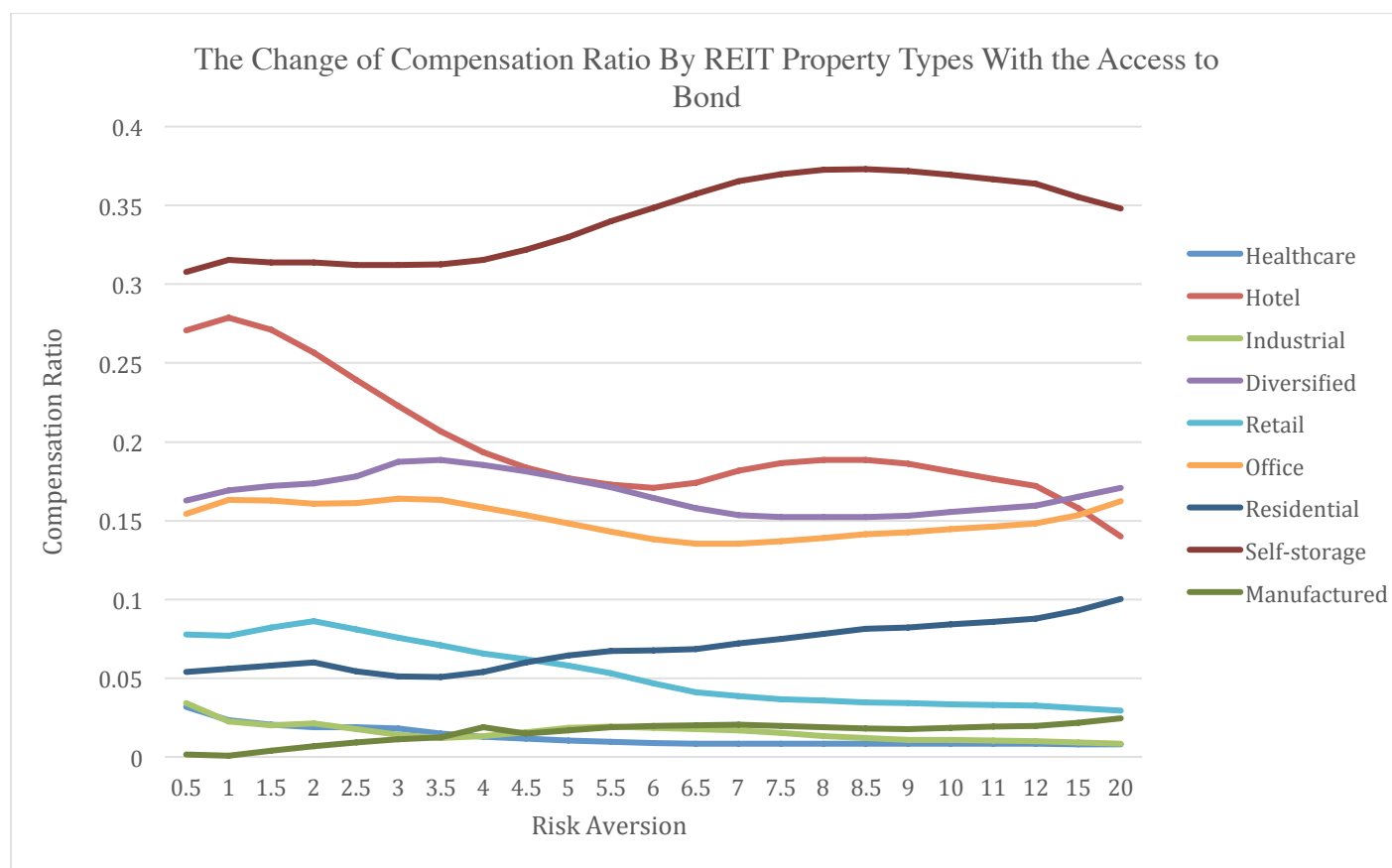REIT Weight in a Portfolio, Long-Only With the Access to Bond
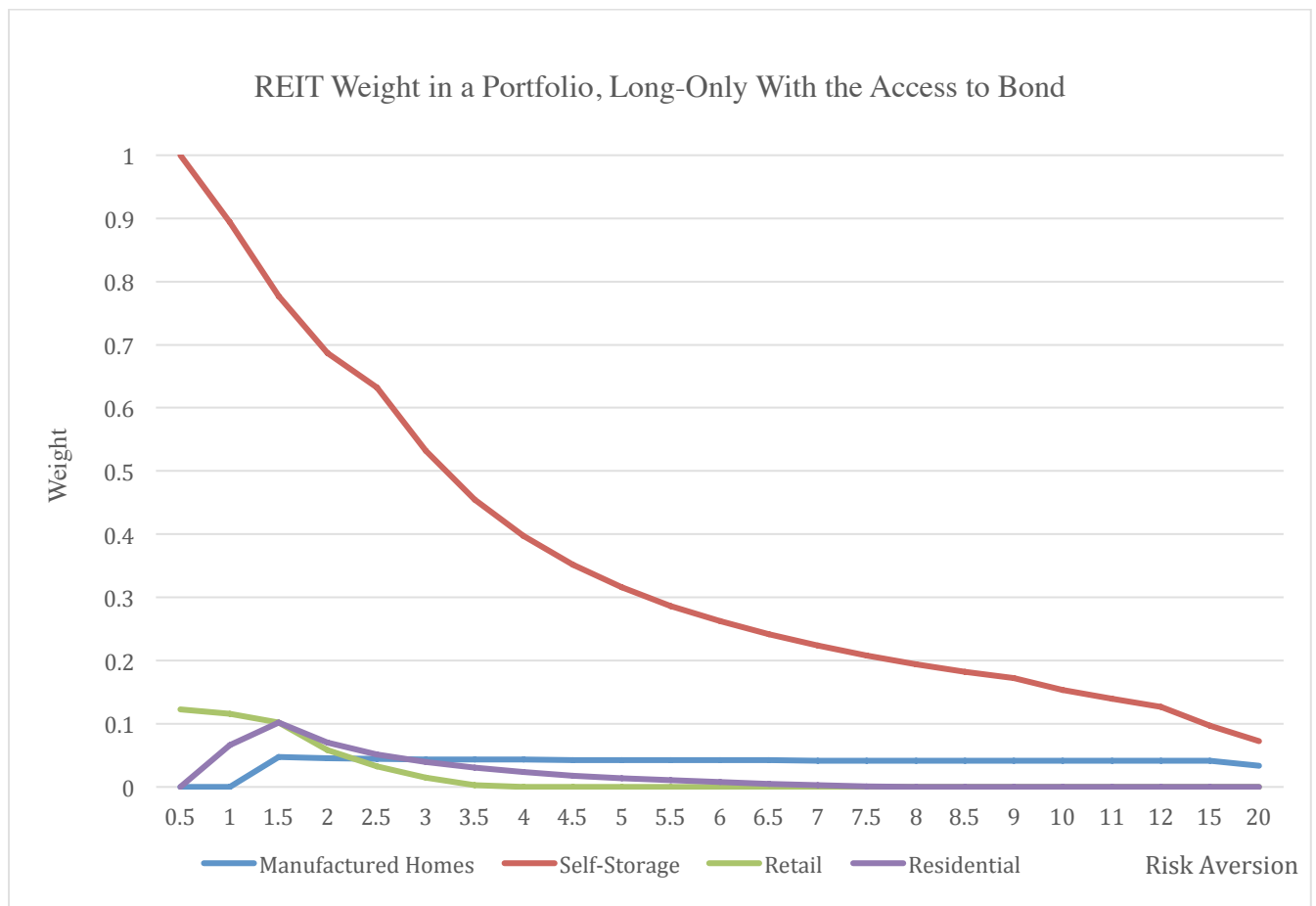
Figure 4.2

Optimal Weights of REITs in Long Only Portfolio Optimization without Bond, By Property Type

Figure 5.1

Compensation Ratio by REIT Property Type, Shorting Selling Allowed, With Bond

The Change of Compensation Ratio By REIT Property Types With the Access to Bond

Figure 5.2

**Compensation Ratio by REIT Property Type, Shorting Selling Allowed,
Without Bond**

The Change of Compensation Ratio By REIT Property Types Without the Access to Bond